

Measurement: Two-Dimensional Figures

9A Perimeter, Circumference, and Area

9-1 Accuracy and Precision

LAB Explore Perimeter and Circumference

9-2 Perimeter and Circumference

LAB Explore Area of Polygons

9-3 Area of Parallelograms

9-4 Area of Triangles and Trapezoids

9-5 Area of Circles

9-6 Area of Irregular Figures

9B Using Squares and Square Roots

LAB Explore Square Roots and Perfect Squares

9-7 Squares and Square Roots

LAB Explore the Pythagorean Theorem

9-8 The Pythagorean Theorem

Ext Identifying and Graphing Irrational Numbers

MULTI-STEP TEST PREP



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Chapter Project Online

KEYWORD: MS7 Ch9

Florida Tropical Fruit Tree Inventory

Tree Type	Number of Trees	Number of Trees per Acre
Grapefruit	14,751,000	181
Lemon	178,800	173
Lime	502,400	159
Orange	84,200,000	128

Career Fruit Tree Grower

Growing fruit trees requires diverse knowledge and skills. A fruit tree grower needs to know how to prepare soil, plant and care for trees, and guard the trees against insects and diseases.

To be successful, a fruit tree grower must also try to maximize the size and quantity of the fruit produced. Growers measure their land to determine the number of trees to plant and where each tree should be planted. The table shows the number and distribution of certain types of fruit trees in Florida.

ARE YOU READY?

✓ Vocabulary

Choose the best term from the list to complete each sentence.

1. A(n) ? is a quadrilateral with exactly one pair of parallel sides.
2. A(n) ? is a four-sided figure with opposite sides that are congruent and parallel.
3. The ? of a circle is one-half the ? of the circle.

diameter
parallelogram
radius
right triangle
trapezoid

Complete these exercises to review skills you will need for this chapter.

✓ Round Whole Numbers

Round each number to the nearest ten and nearest hundred.

4. 1,535
5. 294
6. 30,758
7. 497

✓ Round Decimals

Round each number to the nearest whole number and nearest tenth.

8. 6.18
9. 10.50
10. 513.93
11. 29.06

✓ Multiply with Decimals

Multiply.

12. $5.63 \cdot 8$
13. $9.67 \cdot 4.3$
14. $8.34 \cdot 16$
15. $6.08 \cdot 0.56$
16. $0.82 \cdot 21$
17. $2.74 \cdot 6.6$
18. $40 \cdot 9.54$
19. $0.33 \cdot 0.08$

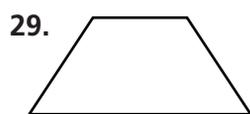
✓ Order of Operations

Simplify each expression.

20. $2 \cdot 9 + 2 \cdot 6$
21. $2(15 + 8)$
22. $4 \cdot 6.8 + 7 \cdot 9.3$
23. $14(25.9 + 13.6)$
24. $(27.3 + 0.7) \div 2^2$
25. $5 \cdot 3^3 - 8.02$
26. $(63 \div 7) \cdot 4^2$
27. $1.1 + 3 \cdot 4.3$
28. $66 \cdot [5 + (3 + 3)^2]$

✓ Identify Polygons

Name each figure.



Where You've Been

Previously, you

- found the perimeter or circumference of geometric figures.
- explored customary and metric units of measure.
- used proportions to convert measurements within the customary system and within the metric system.

In This Chapter

You will study

- comparing perimeter and circumference with the area of geometric figures.
- finding the area of parallelograms, triangles, trapezoids, and circles.
- finding the area of irregular figures.
- using powers, roots, and the Pythagorean Theorem to find missing measures.

Where You're Going

You can use the skills learned in this chapter

- to create an architectural floor plan.
- to design a building access ramp that meets government regulations.

Key Vocabulary/Vocabulario

area	área
circumference	circunferencia
hypotenuse	hipotenusa
perfect square	cuadrado perfecto
perimeter	perímetro
Pythagorean Theorem	teorema de Pitágoras
significant digits	dígitos significativos
square root	raíz cuadrada

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

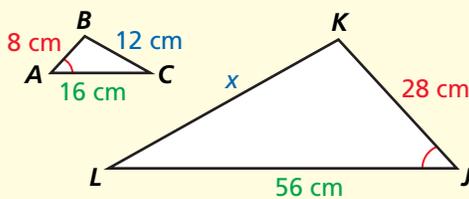
1. The *square root* of a number is one of the two equal factors of the number. For example, 3 is a square root because $3 \cdot 3 = 9$. How might picturing plant roots help you remember the meaning of **square root**?
2. The word *perimeter* comes from the Greek roots *peri*, meaning “all around,” and *metron*, meaning “measure.” What do the Greek roots tell you about the **perimeter** of a geometric figure?
3. To *square a number* means “to multiply the number by itself,” as in $2 \cdot 2$. Keeping this idea of *square* in mind, what do you think a **perfect square** might be?
4. The word *circumference* comes from the Latin word *circumferre*, meaning “to carry around.” How does the Latin meaning help you define the **circumference** of a circle?

Reading Strategy: Read and Interpret Graphics

Figures, diagrams, tables, and graphs provide important data. Knowing how to read these graphics will help you understand and solve related problems.

Similar Figures

$\triangle ABC$ and $\triangle JKL$ are similar.



How to Read

Read all labels.

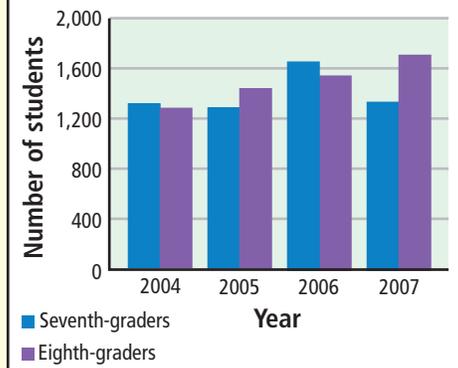
$AB = 8$ cm; $AC = 16$ cm; $BC = 12$ cm;
 $JK = 28$ cm; $JL = 56$ cm; $KL = x$ cm;
 $\angle A$ corresponds to $\angle J$.

Be careful about what you assume.

You may think \overline{AB} corresponds to \overline{LK} , but this is not so. Since $\angle A$ corresponds to $\angle J$, you know \overline{AB} corresponds to \overline{JK} .

Double-Bar Graph

Student Enrollment



How to Read

Read the title of the graph and any special notes.

Blue indicates seventh-graders.
Purple indicates eighth-graders.

Read each axis label and note the intervals of each scale.

x-axis—year increases by 1.

y-axis—enrollment increases by 400 students.

Determine what information is presented.
student enrollment for seventh- and eighth-graders per year

Try This

Look up each graphic in your textbook and answer the following questions.

- Lesson 5-7 Exercise 1: Which side of the smaller triangle corresponds to \overline{BC} ? Can you assume that $\angle BAC$ and $\angle EDF$ are right angles? Explain.
- Lesson 7-3 Example 1: By what interval does the x -axis scale increase? About how many people speak Hindi?

9-1 Accuracy and Precision

Learn to compare the precision of measurements and to determine acceptable levels of accuracy.

Vocabulary

precision

accuracy

significant digits

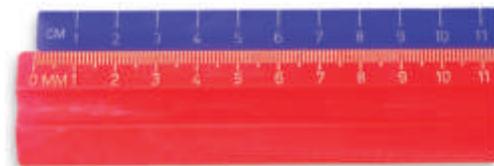
Ancient Greeks used measurements taken during lunar eclipses to determine that the Moon was 240,000 miles from Earth. In 1969, the distance was measured as 221,463 miles.

There is a difference between these measurements because modern scientists conducted the measurement with greater *precision*. **Precision** is the level of detail an instrument can measure.

The smaller the unit an instrument can measure, the more precise its measurements will be. For example, a millimeter ruler has greater precision than a centimeter ruler because it can measure smaller units.



At the University of Texas McDonald Observatory, a laser is used to measure the distance from Earth to the Moon.



EXAMPLE 1 Judging Precision of Measurements

Choose the more precise measurement in each pair.

A 37 in., 3 ft

Since an inch is a smaller unit than a foot, 37 in. is more precise.

B 5 km, 5.8 km

Since tenths are smaller than ones, 5.8 km is more precise.

In the real world, no measurement is exact. The relative exactness of a measurement is its **accuracy**. In a measured value, all the digits that are known with certainty are called **significant digits**. Zeros at the end of a whole number are assumed to be nonsignificant. The table shows the rules for identifying significant digits.

Rule	Example	Number of Significant Digits
• Nonzero digits	45.7	3 significant digits
• Zeros between significant digits	78,002	5 significant digits
• Zeros after the last nonzero digit and to the right of a decimal point	0.0040	2 significant digits

EXAMPLE 2 Identifying Significant Digits

Determine the number of significant digits in each measurement.

A 120.1 mi

The digits 1 and 2 are nonzero digits, and 0 is between two nonzero digits.

So 120.1 mi has 4 significant digits.

B 0.0350 kg

The digits 3 and 5 are nonzero digits, and 0 is to the right of the decimal after the last nonzero digit.

So 0.0350 kg has 3 significant digits.

When you are adding and subtracting measurements, the answer should have the same number of digits to the right of the decimal point as the measurement with the least number of digits to the right of the decimal point.

EXAMPLE 3 Using Significant Digits in Addition or Subtraction

Calculate $45 \text{ mi} - 0.9 \text{ mi}$. Use the correct number of significant digits in the answer.

$$\begin{array}{r} 45 \\ - 0.9 \\ \hline 44.1 \approx 44 \text{ mi} \end{array}$$

0 digits to the right of the decimal point
1 digit to the right of the decimal point
Round the difference so that it has no digits to the right of the decimal point.

When you are multiplying and dividing measurements, the answer must have the same number of significant digits as the measurement with the least number of significant digits.

EXAMPLE 4 Using Significant Digits in Multiplication or Division

Calculate $32.8 \text{ m} \cdot 1.5 \text{ m}$. Use the correct number of significant digits in the answer.

$$\begin{array}{r} 32.8 \\ \times 1.5 \\ \hline 49.2 \approx 49 \text{ m}^2 \end{array}$$

3 significant digits
2 significant digits
Round the product so that it has 2 significant digits.

Think and Discuss

- Tell** how many significant digits there are in 380.102.
- Choose** the more precise measurement: 18 oz or 1 lb. Explain.

GUIDED PRACTICE

See Example 1 Choose the more precise measurement in each pair.

1. 4 ft, 1 yd

2. 2 cm, 21 mm

3. $5\frac{1}{2}$ in., $5\frac{1}{4}$ in.

See Example 2 Determine the number of significant digits in each measurement.

4. 2.703 g

5. 0.02 km

6. 28,000 lb

See Example 3 Calculate. Use the correct number of significant digits in each answer.

7. $16 - 3.8$

8. $3.5 + 0.66$

9. $11.3 - 4$

See Example 4

10. $47.9 \cdot 3.8$

11. $7.0 \cdot 3.6$

12. $50.2 \div 8.0$

INDEPENDENT PRACTICE

See Example 1 Choose the more precise measurement in each pair.

13. 11 in., 1 ft

14. 7.2 m, 6.2 cm

15. 14.2 km, 14 km

16. $4\frac{3}{8}$ in., $4\frac{7}{16}$ in.

17. 2.8 m, 3 m

18. 37 g, 37.0 g

See Example 2 Determine the number of significant digits in each measurement.

19. 0.00002 kg

20. 10,000,000 lb

21. 200.060 m

22. 4.003 L

23. 0.230 cm

24. 940.0 ft

See Example 3 Calculate. Use the correct number of significant digits in each answer.

25. $6.2 + 8.93$

26. $7.02 + 15$

27. $8 - 6.6$

28. $29.1 - 13.204$

29. $8.6 + 9.43$

30. $43.5 + 876.23$

See Example 4

31. $17 \cdot 104$

32. $21.8 \cdot 10.9$

33. $7.0 \div 3.11$

34. $1,680 \div 5.025$

35. $14.2 \div 0.05$

36. $5.22 \cdot 6.3$

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page 744.

Which unit is more precise?

37. foot or mile

38. centimeter or millimeter

39. liter or milliliter

40. minute or second

Calculate. Use the correct number of significant digits in each answer.

41. $38,000 \cdot 4.8$

42. $2.879 + 113.6$

43. $290 - 6.1$

44. $5.6 \div 0.6$

45. $40.29 - 18.5$

46. $24 \div 6.02$

47. **Multi-Step** Jay estimates that he walks 15 miles each week. He walks 1.55 miles to school and then 0.4 miles to his aunt's house after school.

a. Is Jay's estimate reasonable? Explain.

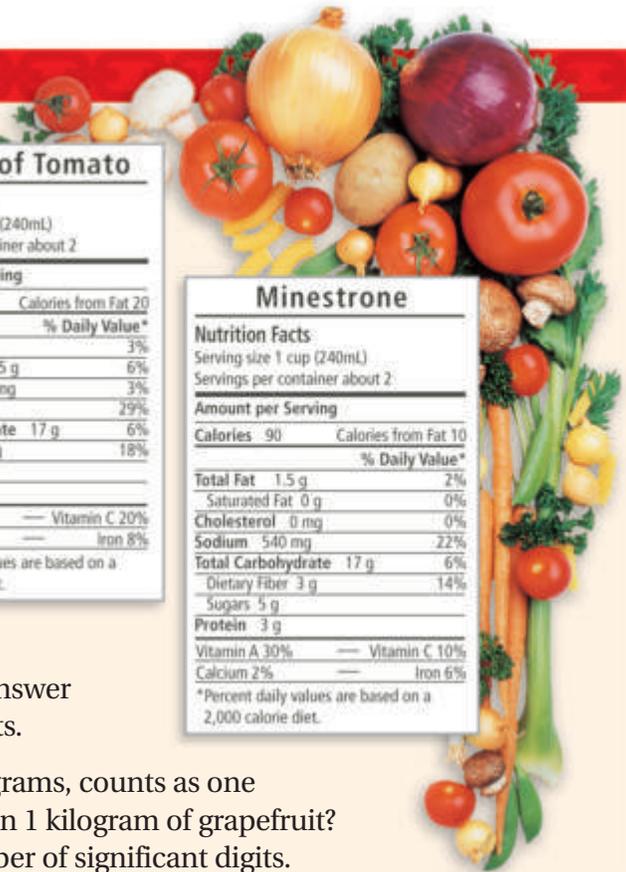
b. How many miles does Jay walk during a 5-day school week? Use the correct number of significant digits in your answer.

The food labels at right give information about two types of soup: cream of tomato and minestrone. Use the labels for Exercises 48 and 49.

48. Which measurement is more precise, the total amount of fat in cream of tomato soup or the total amount in minestrone? Explain.
49. One serving of cream of tomato soup contains 29% of the recommended daily value of sodium for a 2,000-calorie diet. What is the recommended daily value for sodium, in milligrams? Express your answer with the appropriate number of significant digits.
50. One-half of a medium-sized grapefruit, or 154 grams, counts as one serving of fruit. How many servings of fruit are in 1 kilogram of grapefruit? Express your answer with the appropriate number of significant digits.
51.  **Challenge** The greatest possible error of any measurement is half of the smallest unit used in the measurement. For example, 1 pt of juice may actually measure between $\frac{1}{2}$ pt and $1\frac{1}{2}$ pt. What is the range of possible actual weights for a watermelon that was weighed at $19\frac{1}{4}$ lb?

Cream of Tomato	
Nutrition Facts	
Serving size 1 cup (240mL)	
Servings per container about 2	
Amount per Serving	
Calories 100	Calories from Fat 20
% Daily Value*	
Total Fat 2 g	3%
Saturated Fat 1.5 g	6%
Cholesterol 10 mg	3%
Sodium 690 mg	29%
Total Carbohydrate 17 g	6%
Dietary Fiber 4 g	18%
Sugars 11 g	
Protein 2 g	
Vitamin A 20%	Vitamin C 20%
Calcium 0%	Iron 8%
*Percent daily values are based on a 2,000 calorie diet.	

Minestrone	
Nutrition Facts	
Serving size 1 cup (240mL)	
Servings per container about 2	
Amount per Serving	
Calories 90	Calories from Fat 10
% Daily Value*	
Total Fat 1.5 g	2%
Saturated Fat 0 g	0%
Cholesterol 0 mg	0%
Sodium 540 mg	22%
Total Carbohydrate 17 g	6%
Dietary Fiber 3 g	14%
Sugars 5 g	
Protein 3 g	
Vitamin A 30%	Vitamin C 10%
Calcium 2%	Iron 6%
*Percent daily values are based on a 2,000 calorie diet.	



TEST PREP and Spiral Review

52. **Multiple Choice** Which is the most precise measurement?

(A) 1 mile	(B) 1,758 yards	(C) 5,281 feet	(D) 63,355 inches
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53. **Multiple Choice** Which measurement does NOT have three significant digits?

(F) 63.2 cm	(G) 0.08 ft	(H) 0.00500 m	(J) 4.06 yd
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For Exercises 54–56, tell whether you would expect a positive correlation, a negative correlation, or no correlation. (Lesson 7-8)

54. the price of a car and the number of windows it has
55. the speed a car travels and the amount of time it takes to go 100 miles
56. the price per gallon of gasoline and the cost for a tank of gas

Determine whether each figure is a polygon. If it is not, explain why. (Lesson 8-5)





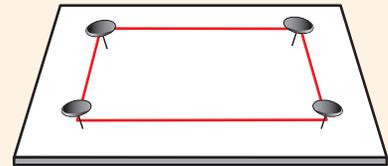
Explore Perimeter & Circumference



The distance around a figure is its perimeter. You can use a loop of string to explore the dimensions of a rectangle with a perimeter of 18 inches.

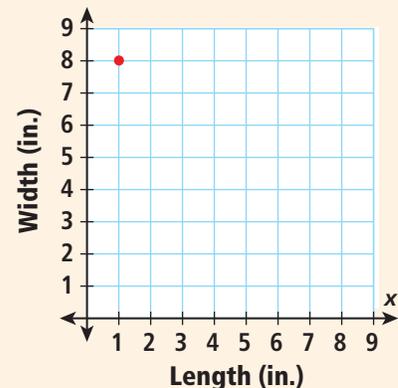
Activity 1

- 1 Cut a piece of string that is slightly longer than 18 inches. Tie the ends together to form an 18-inch loop.
- 2 Make the loop into a rectangle by placing it around four push pins on a corkboard. Both the length and the width of the rectangle should be a whole number of inches.
- 3 Make different rectangles with whole-number lengths and widths. Record the lengths and widths in a table.



Length (in.)	1	2	3	■	■	■	■	■
Width (in.)	8	■	■	■	■	■	■	■

- 4 Graph the data in your table by plotting points on a coordinate plane like the one shown.

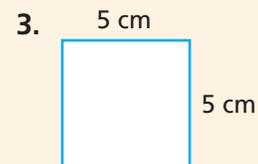
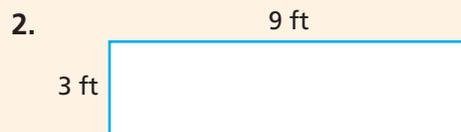
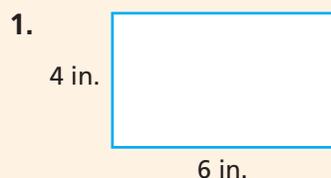


Think and Discuss

1. What pattern is made by the points on your graph?
2. How is the sum of the length and width of each rectangle related to the rectangle's perimeter of 18 inches?
3. Suppose a rectangle has length ℓ and width w . Write a rule that you can use to find the rectangle's perimeter.

Try This

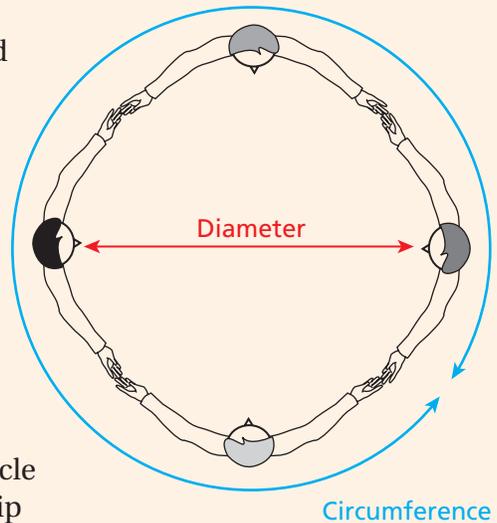
Use the rule you discovered to find the perimeter of each rectangle.



The perimeter of a circle is called the *circumference*. You can explore the relationship between a circle's circumference and its diameter by measuring some circles.

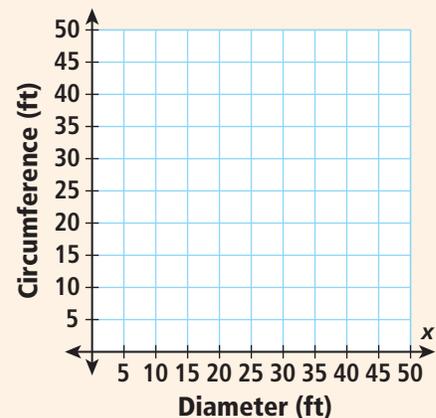
Activity 2

- Four students should stand in a circle with their arms outstretched, as shown in the diagram.
- Another student should find the diameter of the circle by measuring the distance across the middle of the circle with a tape measure.
- The student should also find the circumference of the circle by measuring the distance around the circle from fingertip to fingertip across the backs of the students.
- Record the diameter and circumference in a table like the one below.



Diameter (ft)	■	■	■	■	■
Circumference (ft)	■	■	■	■	■

- Add one or more students to the circle and repeat the process. Record the diameter and circumference for at least five different circles.
- Graph the data in your table by plotting points on a coordinate plane like the one shown.

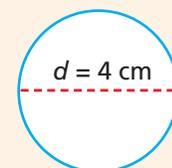


Think and Discuss

- In general, what do you notice about the points on your graph? What shape or pattern do they seem to form?
- Calculate the ratio of the circumference to the diameter for each of the data points. Then calculate the mean of these ratios. For any circle, the ratio of the circumference to the diameter is a constant, known as π (π). Give an estimate for π based on your findings.

Try This

- For a circle with circumference C and diameter d , the ratio of the circumference to the diameter is $\frac{C}{d} = \pi$. Use this to write a formula that you can use to find the circumference of a circle when you know its diameter.
- Use your estimate for the value of π to find the approximate circumference of the circle at right.



9-2

Perimeter and Circumference

Learn to find the perimeter of a polygon and the circumference of a circle.

Vocabulary

perimeter

circumference

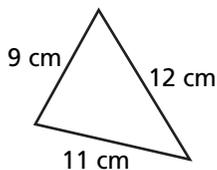
In volleyball, the player serving must hit the ball over the net but keep it within the court's sidelines and end lines. The two sidelines on a volleyball court are each 18 meters long, and the two end lines are each 9 meters long. Together, the four lines form the *perimeter* of the court.

Perimeter is the distance around a geometric figure. To find the perimeter P of a rectangular volleyball court, you can add the lengths of its sides.



EXAMPLE 1 Finding the Perimeter of a Polygon

Find the perimeter.



$$P = 9 + 12 + 11$$

$$P = 32$$

*Use the side lengths.
Add.*

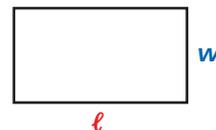
The perimeter of the triangle is 32 cm.

Since opposite sides of a rectangle are equal in length, you can find the perimeter of a rectangle by using a formula.

PERIMETER OF A RECTANGLE

The perimeter P of a rectangle is the sum of twice its length ℓ and twice its width w .

$$P = 2\ell + 2w$$



EXAMPLE 2 Using Properties of a Rectangle to Find Perimeter

Find the perimeter.



$$P = 2\ell + 2w$$

$$P = (2 \cdot 32) + (2 \cdot 15)$$

$$P = 64 + 30$$

$$P = 94$$

Use the formula.

Substitute for \ell and w.

Multiply.

Add.

The perimeter of the rectangle is 94 m.

The distance around a circle is called **circumference**. For every circle, the ratio of circumference C to diameter d is the same. This ratio, $\frac{C}{d}$, is represented by the Greek letter π , called *pi*. *Pi* is approximately equal to 3.14 or $\frac{22}{7}$. By solving the equation $\frac{C}{d} = \pi$ for C , you get the formula for circumference.

CIRCUMFERENCE OF A CIRCLE		
The circumference C of a circle is π times the diameter d , or 2π times the radius r :	$C = \pi d$ or $C = 2\pi r$	

EXAMPLE 3 Finding the Circumference of a Circle

Helpful Hint

If the diameter or radius of a circle is a multiple of 7, use $\frac{22}{7}$ for π .

Find the circumference of each circle to the nearest tenth, if necessary. Use 3.14 or $\frac{22}{7}$ for π .

A

8 in.

$$C = \pi d$$

$$C \approx 3.14 \cdot 8$$

$$C \approx 25.12$$

You know the diameter.
Substitute 3.14 for π and 8 for d .
Multiply.

The circumference of the circle is about 25.1 in.

B

14 cm

$$C = 2\pi r$$

$$C \approx 2 \cdot \frac{22}{7} \cdot 14$$

$$C \approx 88$$

You know the radius.
Substitute $\frac{22}{7}$ for π and 14 for r .
Multiply.

The circumference of the circle is about 88 cm.

EXAMPLE 4 Design Application

Lily is drawing plans for a circular fountain. The circumference of the fountain is 63 ft. What is its approximate diameter?

$$C = \pi d$$

You know the circumference.

$$63 \approx 3.14 \cdot d$$

Substitute 3.14 for π and 63 for C .

$$\frac{63}{3.14} \approx \frac{3.14 \cdot d}{3.14}$$

Divide both sides by 3.14 to isolate the variable.

$$20 \approx d$$

The diameter of the fountain is about 20 ft.

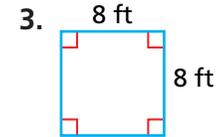
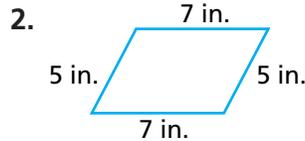
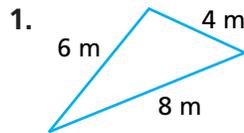
Think and Discuss

- Describe** two ways to find the perimeter of a volleyball court.
- Explain** how to use the formula $C = \pi d$ to find the circumference of a circle if you know the radius.

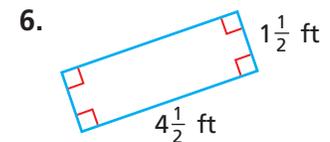
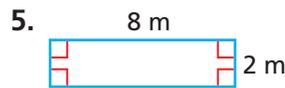
GUIDED PRACTICE

Find each perimeter.

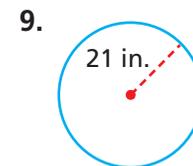
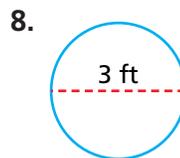
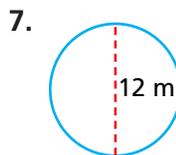
See Example 1



See Example 2



See Example 3

Find the circumference of each circle to the nearest tenth, if necessary. Use 3.14 or $\frac{22}{7}$ for π .

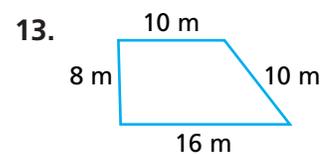
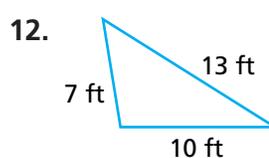
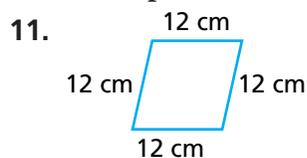
See Example 4

10. A Ferris wheel has a circumference of 440 feet. What is the approximate diameter of the Ferris wheel? Use 3.14 for π .

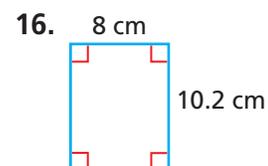
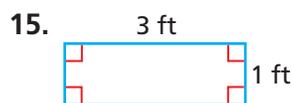
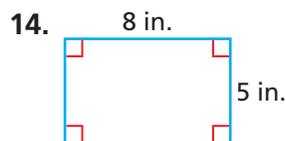
INDEPENDENT PRACTICE

Find each perimeter.

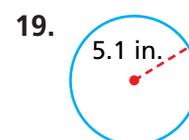
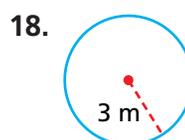
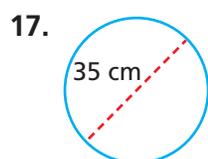
See Example 1



See Example 2



See Example 3

Find the circumference of each circle to the nearest tenth, if necessary. Use 3.14 or $\frac{22}{7}$ for π .

See Example 4

20. The circumference of Kayla's bicycle wheel is 91 inches. What is the approximate diameter of her bicycle wheel? Use 3.14 for π .

PRACTICE AND PROBLEM SOLVING

Extra Practice

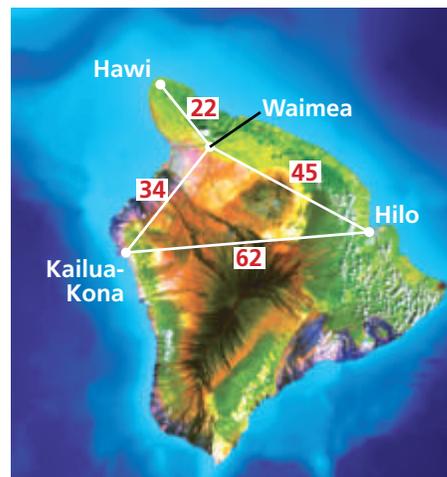
See page 744.

Find each missing measurement to the nearest tenth. Use 3.14 for π .

21. $r = \square$; $d = \square$; $C = 17.8$ m 22. $r = 6.7$ yd; $d = \square$; $C = \square$
23. $r = \square$; $d = 10.6$ in.; $C = \square$ 24. $r = \square$; $d = \square$; $C = \pi$

25. **Critical Thinking** Ben is placing rope lights around the edge of a circular patio with a 24.2 ft diameter. The lights are in lengths of 57 inches. How many strands of lights does he need to surround the patio edge?

26. **Geography** The map shows the distances in miles between the airports on the Big Island of Hawaii. A pilot flies from Kailua-Kona to Waimea to Hilo and back to Kailua-Kona. How far does he travel?



27. **Architecture** The Capitol Rotunda connects the House and Senate sides of the U.S. Capitol. The rotunda is 180 feet tall and has a circumference of about 301.5 feet. What is its approximate diameter, to the nearest foot?

28. **Write a Problem** Write a problem about finding the perimeter or circumference of an object in your school or classroom.
29. **Write About It** Explain how to find the width of a rectangle if you know its perimeter and length.
30. **Challenge** The perimeter of a regular nonagon is $25\frac{1}{2}$ in. What is the length of one side of the nonagon?



TEST PREP and Spiral Review

31. **Multiple Choice** Which is the best estimate for the circumference of a circle with a diameter of 15 inches?
- (A) 18.1 inches (B) 23.6 inches (C) 32.5 inches (D) 47.1 inches
32. **Multiple Choice** John is building a dog pen that is 6 feet by 8 feet. How much fencing material will he need to go all the way around the pen?
- (F) 48 feet (G) 28 feet (H) 20 feet (J) 14 feet

Solve. (Lesson 6-5)

33. 18 is 20% of what number? 34. 78% of 65 is what number?

Calculate. Use the correct number of significant digits in each answer. (Lesson 9-1)

35. $5.8 + 3.27$ 36. $6 - 2.5$ 37. $22.3 \cdot 6.2$ 38. $60.6 \div 15$



Explore Area of Polygons

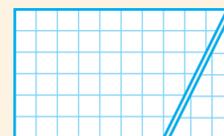
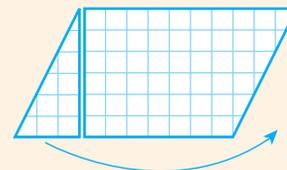
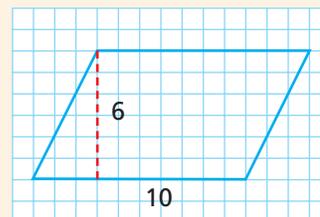
Use with Lessons 9-3, 9-4, and 9-5



You can use a parallelogram to find the area of a triangle or a trapezoid. To do so, you must first know how to find the area of a parallelogram.

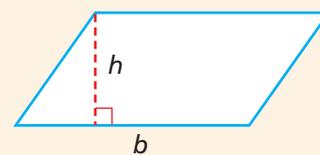
Activity 1

- 1 On a sheet of graph paper, draw a parallelogram with a base of 10 units and a height of 6 units.
- 2 Cut out the parallelogram. Then cut a right triangle off the end of the parallelogram by cutting along the altitude.
- 3 Move the triangle to the other side of the figure to make a rectangle.
- 4 How is the area of the parallelogram related to the area of the rectangle?
- 5 What are the length and width of the rectangle? What is the area of the rectangle?
- 6 Find the area of the parallelogram.



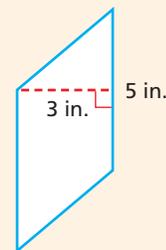
Think and Discuss

1. How are the length and width of the rectangle related to the base and height of the parallelogram?
2. Suppose a parallelogram has base b and height h . Write a formula for the area of the parallelogram.



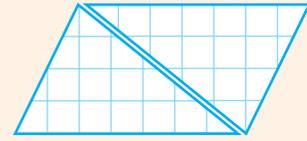
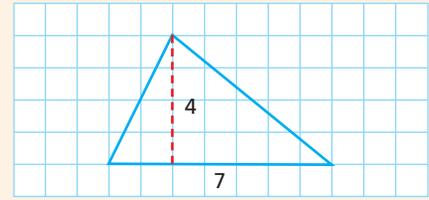
Try This

1. Does your formula work for any parallelogram? If so, show how to use the formula to find the area of the parallelogram at right.
2. Explain what must be true about the areas of the parallelograms below.



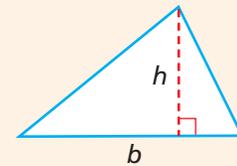
Activity 2

- 1 On a sheet of graph paper, draw a triangle with a base of 7 units and a height of 4 units.
- 2 Cut out the triangle. Then use the triangle to trace and cut out a second triangle that is congruent to it.
- 3 Arrange the two triangles to form a parallelogram.
- 4 How is the area of the triangle related to the area of the parallelogram?
- 5 Find the areas of the parallelogram and the triangle.



Think and Discuss

1. How are the base and height of the triangle related to the base and height of the parallelogram?
2. Suppose a triangle has base b and height h . Write a formula for the area of the triangle.

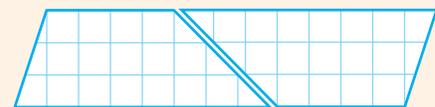
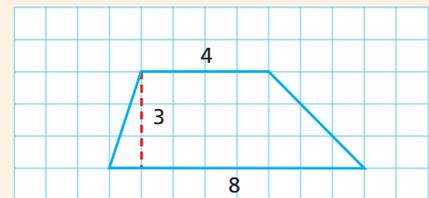


Try This

1. Find the area of a triangle with a base of 10 ft and a height of 5 ft.

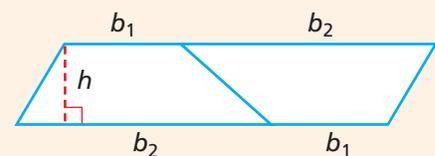
Activity 3

- 1 On a sheet of graph paper, draw a trapezoid with bases 4 units and 8 units long and a height of 3 units.
- 2 Cut out the trapezoid. Then use the trapezoid to trace and cut out a second trapezoid that is congruent to it.
- 3 Arrange the two trapezoids to form a parallelogram.
- 4 How is the area of the trapezoid related to the area of the parallelogram?
- 5 Find the areas of the parallelogram and the trapezoid.



Think and Discuss

1. What is the length of the base of the parallelogram at right? What is the parallelogram's area?
2. What is the area of one of the trapezoids in the figure?



Try This

1. Find the area of a trapezoid with bases 4 in. and 6 in. and a height of 8 in.

9-3

Area of Parallelograms

Learn to find the area of rectangles and other parallelograms.

Vocabulary

area

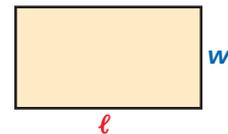
The **area** of a figure is the number of unit squares needed to cover the figure. Area is measured in square units. For example, the area of a chessboard can be measured in square inches. The area of a lawn chessboard is much larger than a regular chessboard, so it can be measured in square feet or square yards.



AREA OF A RECTANGLE

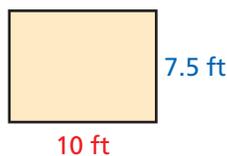
The area A of a rectangle is the product of its length ℓ and its width w .

$$A = \ell w$$



EXAMPLE 1 Finding the Area of a Rectangle

Find the area of the rectangle.



$$A = \ell w$$

$$A = 10 \cdot 7.5$$

$$A = 75$$

The area of the rectangle is 75 ft^2 .

Use the formula.

Substitute for ℓ and w .

Multiply.

EXAMPLE 2 Finding Length or Width of a Rectangle

Bethany and her dad are planting a rectangular garden. The area of the garden is $1,080 \text{ ft}^2$, and the width is 24 ft . What is the length of the garden?

$$A = \ell w \quad \text{Use the formula for the area of a rectangle.}$$

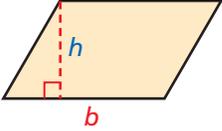
$$1,080 = \ell \cdot 24 \quad \text{Substitute } 1,080 \text{ for } A \text{ and } 24 \text{ for } w.$$

$$\frac{1,080}{24} = \frac{\ell \cdot 24}{24} \quad \text{Divide both sides by } 24 \text{ to isolate } \ell.$$

$$45 = \ell$$

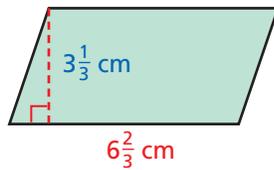
The length of the garden is 45 ft .

The base of a parallelogram is the length of one side. Its height is the perpendicular distance from the base to the opposite side.

AREA OF A PARALLELOGRAM		
The area A of a parallelogram is the product of its base b and its height h .	$A = bh$	

EXAMPLE 3 Finding the Area of a Parallelogram

Find the area of the parallelogram.



$$A = bh$$

Use the formula.

$$A = 6\frac{2}{3} \cdot 3\frac{1}{3}$$

Substitute for b and h .

$$A = \frac{20}{3} \cdot \frac{10}{3}$$

Convert to improper fractions.

$$A = \frac{200}{9} \text{ or } 22\frac{2}{9}$$

Multiply.

The area of the parallelogram is $22\frac{2}{9} \text{ cm}^2$.

EXAMPLE 4 Landscaping Application

Birgit and Mark are building a rectangular patio measuring 9 yd by 7 yd. How many square feet of tile will they need?

First draw and label a diagram. Look at the units. The patio is measured in yards, but the answer should be in square feet.

$$9 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 27 \text{ ft}$$

Convert yards to feet by using a unit conversion factor.



$$7 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 21 \text{ ft}$$

Now find the area of the patio in square feet.

$$A = \ell w$$

Use the formula for the area of a rectangle.

$$A = 27 \cdot 21$$

Substitute 27 for ℓ and 21 for w .

$$A = 567$$

Multiply.

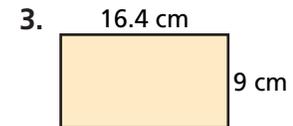
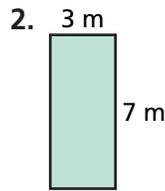
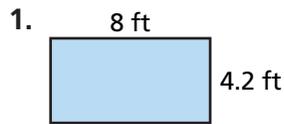
Birgit and Mark need 567 ft^2 of tile.

Think and Discuss

- Write a formula for the area of a square, using an exponent.
- Explain why the area of a nonrectangular parallelogram with side lengths 5 in. and 3 in. is not 15 in^2 .

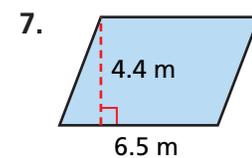
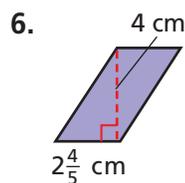
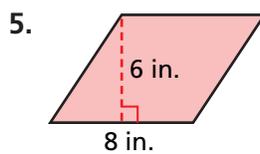
GUIDED PRACTICE

See Example 1 Find the area of each rectangle.



See Example 2 4. Kara wants a rug for her bedroom. She knows the area of her bedroom is 132 ft^2 . The length of her room is 12 ft. What is the width of Kara's bedroom?

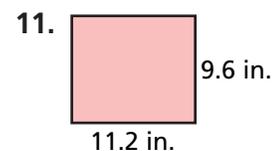
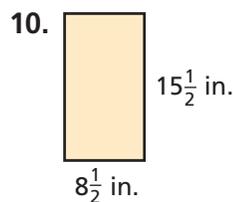
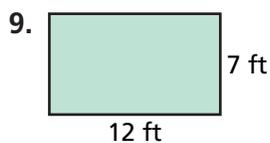
See Example 3 Find the area of each parallelogram.



See Example 4 8. Anna is mowing a rectangular field measuring 120 yd by 66 yd. How many square feet will Anna mow?

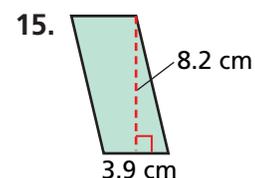
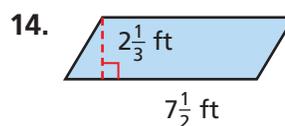
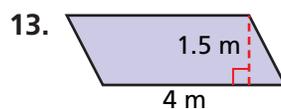
INDEPENDENT PRACTICE

See Example 1 Find the area of each rectangle.



See Example 2 12. James and Linda are fencing a rectangular area of the yard for their dog. The width of the dog yard is 4.5 m. Its area is 67.5 m^2 . What is the length of the dog yard?

See Example 3 Find the area of each parallelogram.



See Example 4 16. Abby is painting rectangular blocks on her bathroom walls. Each block is 15 in. by 18 in. What is the area of one block in square feet?

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page 744.

Find the area of each polygon.

17. rectangle: $\ell = 9$ yd; $w = 8$ yd

18. parallelogram: $b = 7$ m; $h = 4.2$ m

Graph the polygon with the given vertices. Then find the area of the polygon.

19. $(2, 0), (2, -2), (9, 0), (9, -2)$

20. $(4, 1), (4, 7), (8, 4), (8, 10)$

21. **Art** Without the frame, the painting *Girl of Tehuantepec* by Diego Rivera measures about 23 in. by 31 in. The width of the frame is 3 in.

- What is the area of the painting?
- What is the perimeter of the painting?
- What is the total area covered by the painting and the frame?



Girl of Tehuantepec by Diego Rivera

22. What is the height of a parallelogram with an area of 66 in^2 and a base of 11 in.?



23. **Choose a Strategy** The area of a parallelogram is 84 cm^2 . If the base is 5 cm longer than the height, what is the length of the base?

- (A) 5 cm (B) 7 cm (C) 12 cm (D) 14 cm



24. **Write About It** A rectangle and a parallelogram have sides that measure 3 m, 4 m, 3 m, and 4 m. Do the figures have the same area? Explain.



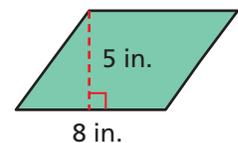
25. **Challenge** Two parallelograms have the same base length, but the height of the first is half that of the second. What is the ratio of the area of the first parallelogram to that of the second? What would the ratio be if both the height and the base of the first parallelogram were half those of the second?



TEST PREP and Spiral Review

26. **Multiple Choice** Find the area of the parallelogram.

- (A) 13 in^2 (B) 26 in^2 (C) 40 in^2 (D) 56 in^2



27. **Extended Response** Kiana is helping her dad build a deck. The plans they have are for a 6-foot-by-8-foot deck, but her dad wants a deck that has twice as much area. He suggests doubling the length of each side of the deck. Will this double the area? If not, suggest another method for doubling the area of the deck.

Tell whether each angle is acute, obtuse, right or straight. (Lesson 8-2)



Find the perimeter of each rectangle, given the dimensions. (Lesson 9-2)

32. 6 in. by 12 in.

33. 2 m by 8 m

34. 16 cm by 3 cm

35. $4\frac{4}{5}$ ft by $1\frac{3}{8}$ ft

9-4

Area of Triangles and Trapezoids

Learn to find the area of triangles and trapezoids.

The Bermuda Triangle is a triangular region between Bermuda, Florida, and Puerto Rico. To find the area of this region, you could use the formula for the area of a triangle.

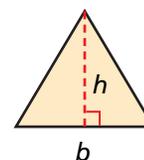
The base of a triangle can be any side. The height of a triangle is the perpendicular distance from the base to the opposite vertex.



AREA OF A TRIANGLE

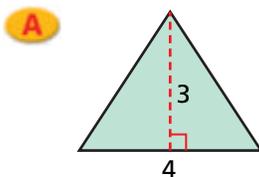
The area A of a triangle is half the product of its base b and its height h .

$$A = \frac{1}{2}bh$$



EXAMPLE 1 Finding the Area of a Triangle

Find the area of each triangle.



$$A = \frac{1}{2}bh$$

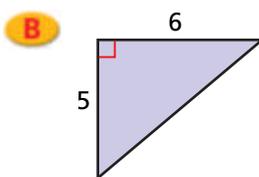
Use the formula.

$$A = \frac{1}{2}(4 \cdot 3)$$

Substitute 4 for b and 3 for h .

$$A = 6$$

The area of the triangle is 6 square units.



$$A = \frac{1}{2}bh$$

Use the formula.

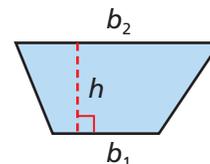
$$A = \frac{1}{2}(6 \cdot 5)$$

Substitute 6 for b and 5 for h .

$$A = 15$$

The area of the triangle is 15 square units.

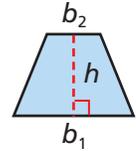
The two parallel sides of a trapezoid are its bases, b_1 and b_2 . The height of a trapezoid is the perpendicular distance between the bases.



AREA OF A TRAPEZOID

The area of a trapezoid is half its height multiplied by the sum of the lengths of its two bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$



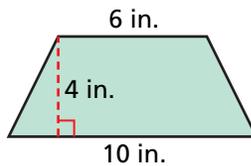
EXAMPLE 2 Finding the Area of a Trapezoid

Reading Math

In the term b_1 , the number 1 is called a *subscript*. It is read as “b-one” or “b sub-one.”

Find the area of each trapezoid.

A



$$A = \frac{1}{2}h(b_1 + b_2)$$

Use the formula.

$$A = \frac{1}{2} \cdot 4(10 + 6)$$

Substitute.

$$A = \frac{1}{2} \cdot 4(16)$$

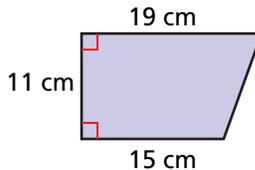
Add.

$$A = 32$$

Multiply.

The area of the trapezoid is 32 in^2 .

B



$$A = \frac{1}{2}h(b_1 + b_2)$$

Use the formula.

$$A = \frac{1}{2} \cdot 11(15 + 19)$$

Substitute.

$$A = \frac{1}{2} \cdot 11(34)$$

Add.

$$A = 187$$

Multiply.

The area of the trapezoid is 187 cm^2 .

EXAMPLE 3 Geography Application

The state of Nevada is shaped somewhat like a trapezoid. What is the approximate area of Nevada?

$$A = \frac{1}{2}h(b_1 + b_2)$$

Use the formula.

$$A = \frac{1}{2} \cdot 320(200 + 475)$$

Substitute.

$$A = \frac{1}{2} \cdot 320(675)$$

Add.

$$A = 108,000$$

Multiply.

The area of Nevada is approximately 108,000 square miles.



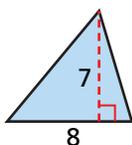
Think and Discuss

- 1. Tell** how to use the sides of a right triangle to find its area.
- 2. Explain** how to find the area of a trapezoid.

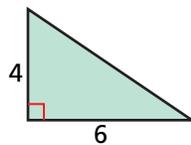
GUIDED PRACTICE

See Example 1 Find the area of each triangle.

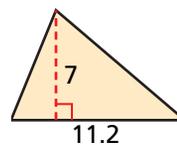
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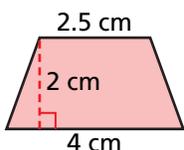


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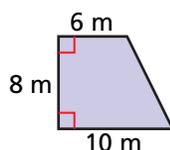


See Example 2 Find the area of each trapezoid.

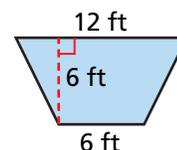
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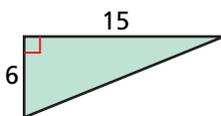
See Example 3 7. The state of Tennessee is shaped somewhat like a parallelogram. What is the approximate area of Tennessee?



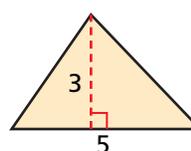
INDEPENDENT PRACTICE

See Example 1 Find the area of each triangle.

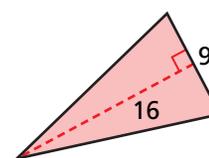
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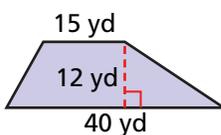


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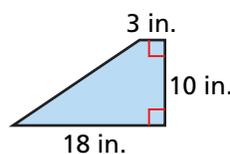


See Example 2 Find the area of each trapezoid.

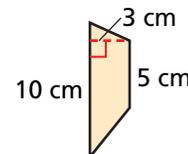
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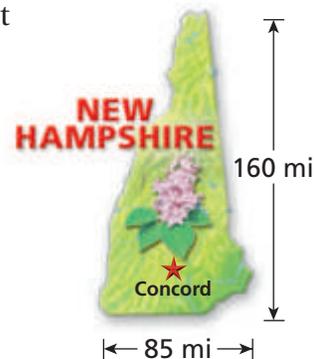
12.



13.



See Example 3 14. The state of New Hampshire is shaped somewhat like a right triangle. What is the approximate area of New Hampshire?



PRACTICE AND PROBLEM SOLVING

Extra Practice

See page 744.

Find the missing measurement of each triangle.

15. $b = 8$ cm
 $h = \square$
 $A = 18$ cm²

16. $b = 16$ ft
 $h = 0.7$ ft
 $A = \square$

17. $b = \square$
 $h = 95$ in.
 $A = 1,045$ in²

Graph the polygon with the given vertices. Then find the area of the polygon.

18. (1, 2), (4, 5), (8, 2), (8, 5)

19. (1, -6), (5, -1), (7, -6)

20. (2, 3), (2, 10), (7, 6), (7, 8)

21. (3, 0), (3, 4), (-3, 0)

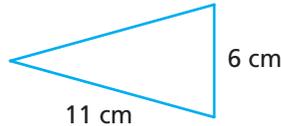
22. What is the height of a trapezoid with an area of 9 m² and bases that measure 2.4 m and 3.6 m?

23. **Multi-Step** The state of Colorado is somewhat rectangular in shape. Estimate the perimeter and area of Colorado.



24. **What's the Error?**

A student says the area of the triangle shown at right is 33 cm². Explain why the student is incorrect.



25. **Write About It** Explain how to use the formulas for the area of a rectangle and the area of a triangle to estimate the area of Nevada.



26. **Challenge** The state of North Dakota is trapezoidal in shape and has an area of 70,704 mi². If the southern border is 359 mi and the distance between the northern border and the southern border is 210 mi, what is the approximate length of the northern border?



TEST PREP and Spiral Review

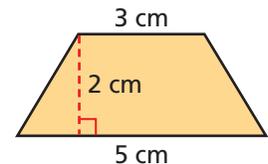
27. **Multiple Choice** Find the area of the trapezoid.

(A) 8 cm²

(C) 17 cm²

(B) 16 cm²

(D) 30 cm²



28. **Short Response** Graph the triangle with vertices (0, 0), (2, 3), and (6, 0). Then find the area of the triangle.

Find the measure of the third angle in each triangle, given two angle measures. (Lesson 8-8)

29. 45°, 45°

30. 71°, 57°

31. 103°, 28°

32. 62°, 19°

33. Justin is laying a tile floor in a room that measures 5 yd by 6 yd. How many square feet of tile does he need? (Lesson 9-3)

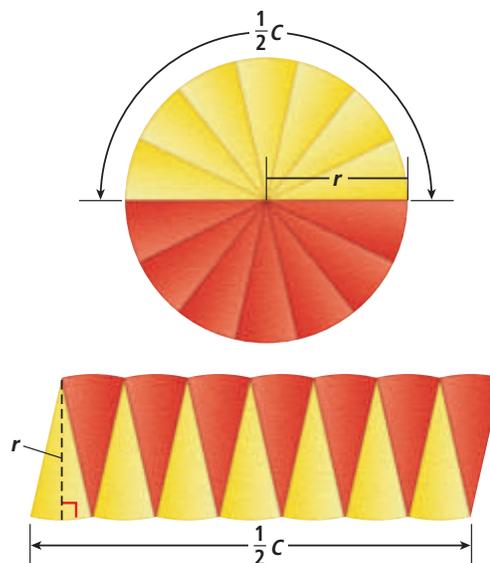
9-5 Area of Circles

Learn to find the area of circles.

A circle can be cut into equal-sized sectors and arranged to resemble a parallelogram. The height h of the parallelogram is equal to the radius r of the circle, and the base b of the parallelogram is equal to one-half the circumference C of the circle. So the area of the parallelogram can be written as

$$A = bh, \text{ or } A = \frac{1}{2}Cr.$$

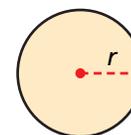
$$\text{Since } C = 2\pi r, A = \frac{1}{2}(2\pi r)r = \pi r^2.$$



AREA OF A CIRCLE

The area A of a circle is the product of π and the square of the circle's radius r .

$$A = \pi r^2$$

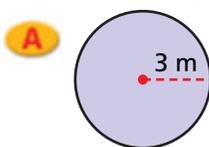


EXAMPLE 1 Finding the Area of a Circle

Find the area of each circle to the nearest tenth. Use 3.14 for π .

Remember!

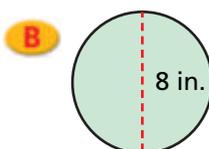
The order of operations calls for evaluating the exponents before multiplying.



$$\begin{aligned} A &= \pi r^2 \\ A &\approx 3.14 \cdot 3^2 \\ A &\approx 3.14 \cdot 9 \\ A &\approx 28.26 \end{aligned}$$

Use the formula.
Substitute. Use 3 for r .
Evaluate the power.
Multiply.

The area of the circle is about 28.3 m².



$$\begin{aligned} A &= \pi r^2 \\ A &\approx 3.14 \cdot 4^2 \\ A &\approx 3.14 \cdot 16 \\ A &\approx 50.24 \end{aligned}$$

Use the formula.
Substitute. Use 4 for r .
Evaluate the power.
Multiply.

The area of the circle is about 50.2 in².

EXAMPLE 2 Social Studies Application

Social Studies



Nomads in Mongolia carried their homes wherever they roamed. These homes, called *yurts*, were made of wood and felt.

A group of historians are building a yurt to display at a local multicultural fair. The yurt has a height of 8 feet 9 inches at its center, and it has a circular floor of radius 7 feet. What is the area of the floor of the yurt? Use $\frac{22}{7}$ for π .

$$A = \pi r^2 \quad \text{Use the formula for the area of a circle.}$$

$$A \approx \frac{22}{7} \cdot 7^2 \quad \text{Substitute. Use 7 for } r.$$

$$A \approx \frac{22}{\cancel{7}_1} \cdot \overset{7}{\cancel{49}} \quad \text{Evaluate the power. Then simplify.}$$

$$A \approx 22 \cdot 7$$

$$A \approx 154 \quad \text{Multiply.}$$

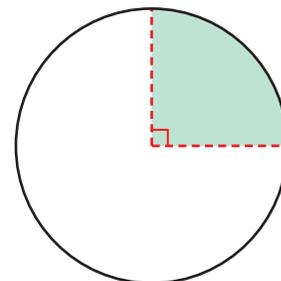
The area of the floor of the yurt is about 154 ft².

EXAMPLE 3 Measurement Application

Helpful Hint

To estimate the area of a circle, you can square the radius and multiply by 3.

Use a centimeter ruler to measure the radius of the circle. Then find the area of the shaded region of the circle. Use 3.14 for π . Round your answer to the nearest tenth.



First measure the radius of the circle: It measures 1.8 cm.

Now find the area of the entire circle.

$$A = \pi r^2 \quad \text{Use the formula for the area of a circle.}$$

$$A \approx 3.14 \cdot 1.8^2 \quad \text{Substitute. Use 1.8 for } r \text{ and 3.14 for } \pi.$$

$$A \approx 3.14 \cdot 3.24 \quad \text{Evaluate the power.}$$

$$A \approx 10.1736 \quad \text{Multiply.}$$

Since $\frac{1}{4}$ of the circle is shaded, divide the area of the circle by 4.

$$10.1736 \div 4 = 2.5434$$

The area of the shaded region of the circle is about 2.5 cm².

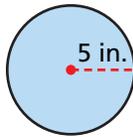
Think and Discuss

- 1. Compare** finding the area of a circle when given the radius with finding the area when given the diameter.
- 2. Give an example** of a circular object in your classroom. Tell how you could estimate the area of the object, and then estimate.

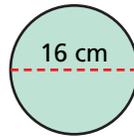
GUIDED PRACTICE

See Example 1 Find the area of each circle to the nearest tenth. Use 3.14 for π .

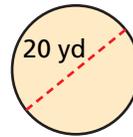
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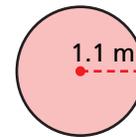
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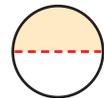


4.



See Example 2 5. The most popular pizza at Sam's Pizza is the 14-inch pepperoni pizza. What is the area of a pizza with a diameter of 14 inches? Use $\frac{22}{7}$ for π .

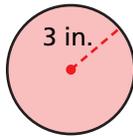
See Example 3 6. **Measurement** Use a centimeter ruler to measure the diameter of the circle. Then find the area of the shaded region of the circle. Use 3.14 for π . Round your answer to the nearest tenth.



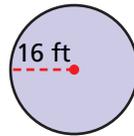
INDEPENDENT PRACTICE

See Example 1 Find the area of each circle to the nearest tenth. Use 3.14 for π .

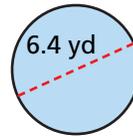
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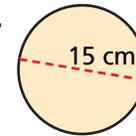
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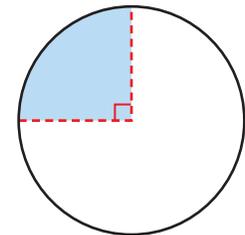


10.



See Example 2 11. A wheel has a radius of 14 centimeters. What is the area of the wheel? Use $\frac{22}{7}$ for π .

See Example 3 12. **Measurement** Use a centimeter ruler to measure the radius of the circle. Then find the area of the shaded region of the circle. Use 3.14 for π . Round your answer to the nearest tenth.



PRACTICE AND PROBLEM SOLVING

Extra Practice

See page 744.

13. A radio station broadcasts a signal over an area with a 75-mile radius. What is the area of the region that receives the radio signal?
14. A circular flower bed in Kay's backyard has a diameter of 8 feet. What is the area of the flower bed? Round your answer to the nearest tenth.
15. A company is manufacturing aluminum lids. The radius of each lid is 3 cm. What is the area of one lid? Round your answer to the nearest tenth.

Given the radius or diameter, find the circumference and area of each circle to the nearest tenth. Use 3.14 for π .

16. $r = 7$ m

17. $d = 18$ in.

18. $d = 24$ ft

19. $r = 6.4$ cm

Given the area, find the radius of each circle. Use 3.14 for π .

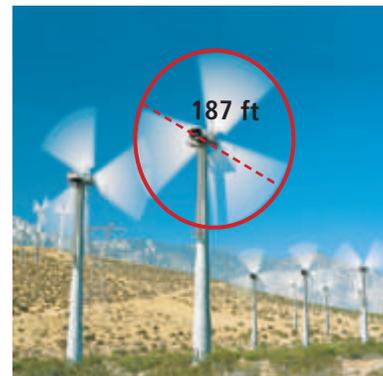
20. $A = 113.04$ cm²

21. $A = 3.14$ ft²

22. $A = 28.26$ in²

23. A hiker was last seen near a fire tower in the Catalina Mountains. Searchers are dispatched to the surrounding area to find the missing hiker.
- Assume the hiker could walk in any direction at a rate of 3 miles per hour. How large an area would searchers have to cover if the hiker was last seen 2 hours ago? Use 3.14 for π . Round your answer to the nearest square mile.
 - How much additional area would the searchers have to cover if the hiker was last seen 3 hours ago?

24. **Physical Science** The tower of a wind turbine is about the height of a 20-story building. Each turbine can produce 24 megawatt-hours of electricity in one day. Find the area covered by the turbine when it is rotating. Use 3.14 for π . Round your answer to the nearest tenth.



25. **Critical Thinking** Two circles have the same radius. Is the combined area of the two circles the same as the area of a circle with twice the radius?

26. **What's the Question?** Chang painted half of a free-throw circle that has a diameter of 12 ft. The answer is 56.52 ft^2 . What is the question?
27. **Write About It** Describe how to find the area of a circle when given only the circumference of the circle.
28. **Challenge** How does the area of a circle change if you multiply the radius by a factor of n , where n is a whole number?

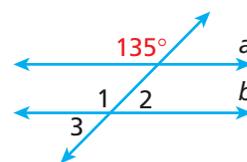


TEST PREP and Spiral Review

29. **Multiple Choice** The area of a circle is 30 square feet. A second circle has a radius that is 2 feet shorter than that of the first circle. What is the area, to the nearest tenth, of the second circle? Use 3.14 for π .
- (A) 3.7 square feet (B) 10.0 square feet (C) 38.0 square feet (D) 179.2 square feet
30. **Short Response** A pizza parlor offers a large pizza with a 12-inch diameter. It also offers a "mega" pizza with a 24-inch diameter. The slogan used to advertise the mega pizza is "Twice the pizza of a large, and twice the fun." Is the mega pizza twice as big as the large? If not, how much bigger is it? Explain.

Line $a \parallel$ line b . Use the diagram to find each angle measure. (Lesson 8-3)

31. $m\angle 1$ 32. $m\angle 2$ 33. $m\angle 3$



Graph the polygon with the given vertices. Then find the area of the polygon. (Lesson 9-4)

34. $(-1, 1), (0, 4), (4, 1)$ 35. $(-3, 3), (2, 3), (1, -1), (-1, -1)$

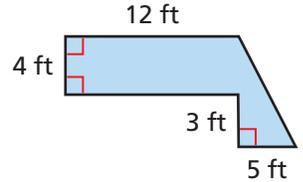
EXAMPLE 3 PROBLEM SOLVING APPLICATION



Helpful Hint

There are often several different ways to separate an irregular figure into familiar figures.

Chandra wants to carpet the floor of her closet. A floor plan of the closet is shown at right. How much carpet does she need?



1 Understand the Problem

Rewrite the question as a statement:

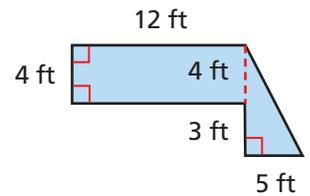
- Find the amount of carpet needed to cover the floor of the closet.

List the **important information**:

- The floor of the closet is an irregular figure.
- The amount of carpet needed is equal to the area of the floor.

2 Make a Plan

Find the area of the floor by separating the figure into familiar figures: a rectangle and a triangle. Then add the areas of the rectangle and triangle to find the total area.



3 Solve

Find the area of each smaller figure.

Area of the rectangle:

$$A = \ell w$$

$$A = 12 \cdot 4$$

$$A = 48 \text{ ft}^2$$

Area of the triangle:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(3 + 4)$$

$$A = \frac{1}{2}(35) = 17.5 \text{ ft}^2$$

Add the areas to find the total area.

$$A = 48 + 17.5 = 65.5$$

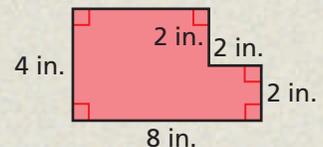
Chandra needs 65.5 ft^2 of carpet.

4 Look Back

The area of the closet floor must be greater than the area of the rectangle (48 ft^2), so the answer is reasonable.

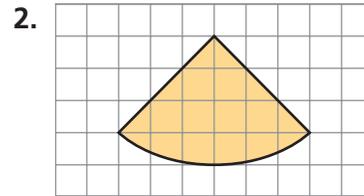
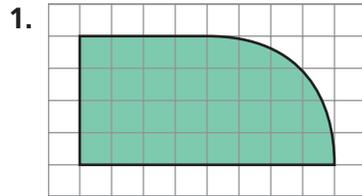
Think and Discuss

1. **Describe** two different ways to find the area of the irregular figure at right.
2. **Explain** why the area of the figure at right must be less than 32 in^2 .

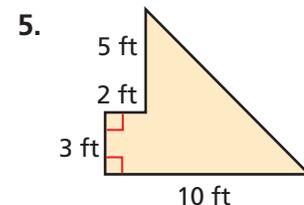
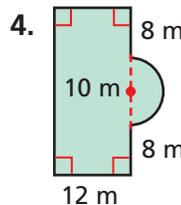
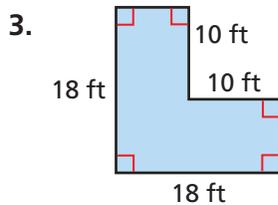


GUIDED PRACTICE

See Example 1 Estimate the area of each figure. Each square represents 1 ft^2 .



See Example 2 Find the area of each figure. Use 3.14 for π .

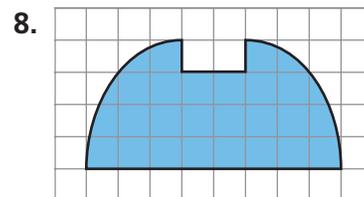
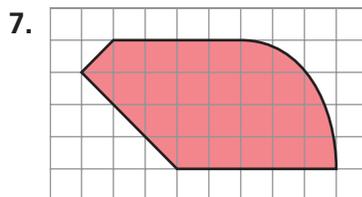


See Example 3 6. Luis has a model train set. The layout of the track is shown at right. How much artificial grass does Luis need in order to fill the interior of the layout? Use 3.14 for π .

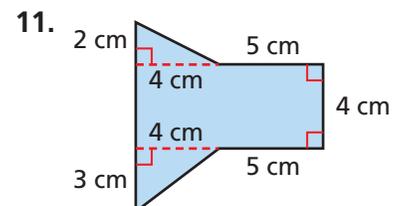
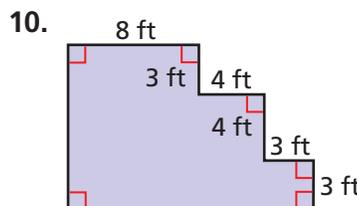
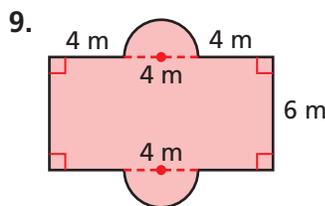


INDEPENDENT PRACTICE

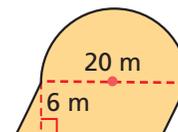
See Example 1 Estimate the area of each figure. Each square represents 1 ft^2 .



See Example 2 Find the area of each figure. Use 3.14 for π .



See Example 3 12. The figure shows the floor plan for a gallery of a museum. The ceiling of the gallery is to be covered with soundproofing material. How much material is needed? Use 3.14 for π .

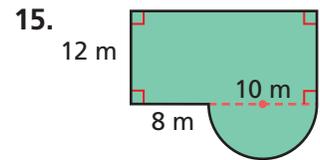
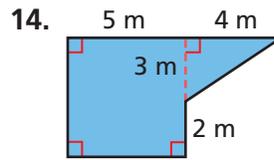
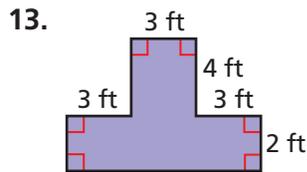


PRACTICE AND PROBLEM SOLVING

Extra Practice

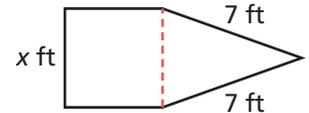
See page 745.

Find the area and perimeter of each figure. Use 3.14 for π .



16. **Multi-Step** A figure has vertices $A(-8, 5)$, $B(-4, 5)$, $C(-4, 2)$, $D(3, 2)$, $E(3, -2)$, $F(6, -2)$, $G(6, -4)$, and $H(-8, -4)$. Graph the figure on a coordinate plane. Then find the area and perimeter of the figure.

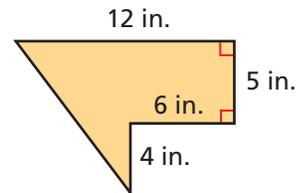
17. **Critical Thinking** The figure at right is made up of an isosceles triangle and a square. The perimeter of the figure is 44 feet. What is the value of x ?



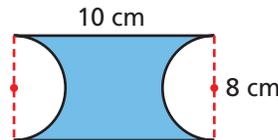
18. **Choose a Strategy** A figure is formed by combining a square and a triangle. Its total area is 32.5 m^2 . The area of the triangle is 7.5 m^2 . What is the length of each side of the square?

- (A) 5 m (B) 15 m (C) 16.25 m (D) 25 m

19. **Write About It** Describe how to find the area of the irregular figure at right.



20. **Challenge** Find the area and perimeter of the figure at right. Use 3.14 for π .

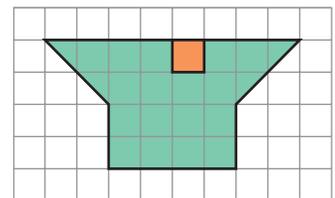


TEST PREP and Spiral Review

21. **Multiple Choice** A rectangle is formed by two congruent right triangles. The area of each triangle is 6 in^2 . Each side of the rectangle is a whole number of inches. Which of these CANNOT be the perimeter of the rectangle?

- (A) 26 in. (B) 24 in. (C) 16 in. (D) 14 in.

22. **Extended Response** The shaded area of the garden represents a patch of carrots. Veronica estimates that she will get about 12 carrots from this patch. Veronica is going to plant the rest of her garden with carrots. Estimate the total number of carrots she can expect to grow.



$\angle 1$ and $\angle 2$ are complementary angles. Find $m\angle 2$. (Lesson 8-2)

23. $m\angle 1 = 33^\circ$ 24. $m\angle 1 = 46^\circ$ 25. $m\angle 1 = 60^\circ$ 26. $m\angle 1 = 25.5^\circ$

Given the diameter, find the area of each circle to the nearest tenth.

Use 3.14 for π . (Lesson 9-5)

27. $d = 30 \text{ m}$ 28. $d = 5.5 \text{ cm}$ 29. $d = 18 \text{ in.}$ 30. $d = 11 \text{ ft}$

Quiz for Lessons 9-1 Through 9-6

 **9-1 Accuracy and Precision**

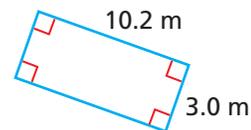
1. Which measurement is more precise—5 in. or 56 ft?

Calculate. Use the correct number of significant digits in each answer.

2. $329 + 640$ 3. $5.6 \cdot 2.59$ 4. $82.5 \div 16$ 5. $27.1 - 4$

 **9-2 Perimeter and Circumference**

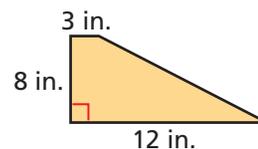
6. Find the perimeter of the figure at right.
7. If the circumference of a wheel is 94 cm, what is its approximate diameter?


 **9-3 Area of Parallelograms**

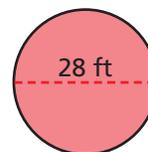
8. The area of a rectangular courtyard is $1,508 \text{ m}^2$, and the length is 52 m. What is the width of the courtyard?
9. Jackson's kitchen is 8 yd by 3 yd. What is the area of his kitchen in square feet?

 **9-4 Area of Triangles and Trapezoids**

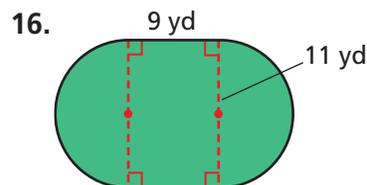
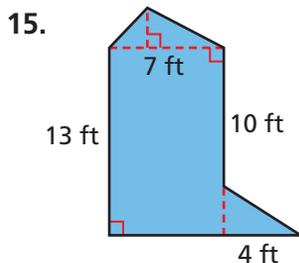
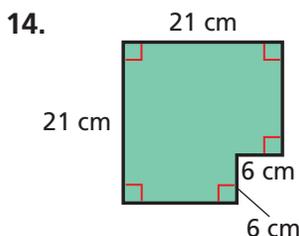
10. Find the area of the trapezoid at right.
11. A triangle has an area of 45 cm^2 and a base of 12.5 cm. What is the height of the triangle?


 **9-5 Area of Circles**

12. Find the area of the circle to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for π .
13. The radius of a clock face is $8\frac{3}{4}$ in. What is its area to the nearest whole number?


 **9-6 Area of Irregular Figures**

Find the area of each figure to the nearest tenth. Use 3.14 for π .



Focus on Problem Solving



Understand the Problem

- Identify too much or too little information

Problems involving real-world situations sometimes give too much or too little information. Before solving these types of problems, you must decide what information is necessary and whether you have all the necessary information.

If the problem gives too much information, identify which of the facts are really needed to solve the problem. If the problem gives too little information, determine what additional information is required to solve the problem.



Copy each problem and underline the information you need to solve it. If necessary information is missing, write down what additional information is required.

- 1 Mrs. Wong wants to put a fence around her garden. One side of her garden measures 8 feet. Another side measures 5 feet. What length of fencing does Mrs. Wong need to enclose her garden?
- 2 Two sides of a triangle measure 17 inches and 13 inches. The perimeter of the triangle is 45 inches. What is the length in feet of the third side of the triangle? (There are 12 inches in 1 foot.)
- 3 During swim practice, Peggy swims 2 laps each of freestyle and backstroke. The dimensions of the pool are 25 meters by 50 meters. What is the area of the pool?
- 4 Each afternoon, Curtis walks his dog two times around the park. The park is a rectangle that is 315 yards long. How far does Curtis walk his dog each afternoon?
- 5 A trapezoid has bases that measure 12 meters and 18 meters and one side that measures 9 meters. The trapezoid has no right angles. What is the area of the trapezoid?





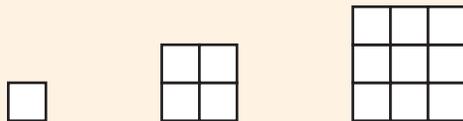
Explore Square Roots and Perfect Squares



You can use geometric models such as tiles or graph paper to represent squares and square roots.

Activity 1

- Copy the three square arrangements below on graph paper. Continue the pattern until you have drawn 10 square arrangements.

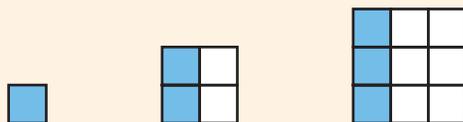


- Copy and complete the table below. In the first column, write the number of small squares in each figure you drew. To complete the second column, use a calculator to find the square root.

(To find the square root of 4, press **2nd** $\sqrt{x^2}$ 4 **)** **ENTER** .)

Total Number of Small Squares	Square Root
1	1
4	2
9	3
■	■
■	■
■	■
■	■
■	■
■	■
■	■

- Shade in one column of each square arrangement that you drew in **1**.



Think and Discuss

1. How does the square root relate to the total number of small squares in a figure?
2. How does the square root in the table relate to the shaded portion of each figure?

Try This

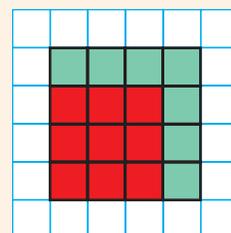
Use graph paper to find each square root.

1. 121
2. 144
3. 196

Activity 2

Follow the steps below to estimate $\sqrt{14}$.

- 1 On graph paper, use one color to draw the smallest possible square arrangement using at least 14 small squares.
- 2 On the same arrangement, draw the largest possible square arrangement using less than 14 small squares.
- 3 Count the number of squares in each arrangement. Notice that 14 is between these numbers.



Number in small arrangement *Number in large arrangement*

$$9 < 14 < 16$$

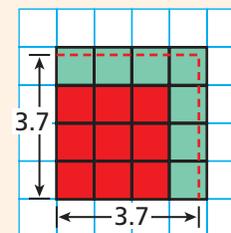
- 4 Use a calculator to find $\sqrt{14}$ to the nearest tenth. $\sqrt{14} = 3.7$. Use inequality symbols to compare the square roots of 9, 14, and 16.

$$\sqrt{9} < \sqrt{14} < \sqrt{16}$$

$$3 < 3.7 < 4$$

The square root of 9 is less than the square root of 14, which is less than the square root of 16.

- 5 Use dashed lines on the figure to sketch a square that is 3.7 units on each side.



Think and Discuss

1. Describe how to use two numbers to estimate the square roots of nonperfect squares without using a calculator.
2. Explain how you can use graph paper to estimate $\sqrt{19}$.
3. Name three numbers that have square roots between 5 and 6.

Try This

Use graph paper to estimate each square root. Then use a calculator to find the square root to the nearest tenth.

1. $\sqrt{19}$
2. $\sqrt{10}$
3. $\sqrt{28}$
4. $\sqrt{35}$

9-7

Squares and Square Roots

Learn to find and estimate square roots of numbers.

Vocabulary

perfect square

square root

radical sign

A square with sides that measure 3 units each has an area of $3 \cdot 3$, or 3^2 . Notice that the area of the square is represented by a power in which the base is the side length and the exponent is 2. A power in which the exponent is 2 is called a *square*.

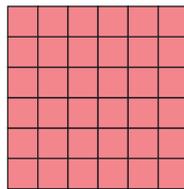


EXAMPLE 1 Finding Squares of Numbers

Find each square.

A 6^2

Method 1: Use a model.



$$A = \ell w$$

$$A = 6 \cdot 6$$

$$A = 36$$

The square of 6 is 36.

B 14^2

Method 2: Use a calculator.

Press 14  .

$$14^2 = 196$$

The square of 14 is 196.

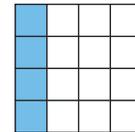
Reading Math



$\sqrt{16} = 4$ is read as "The square root of 16 is 4."

A **perfect square** is the square of a whole number. The number 36 is a perfect square because $36 = 6^2$ and 6 is a whole number.

The **square root** of a number is one of the two equal factors of the number. Four is a square root of 16 because $4 \cdot 4 = 16$. The symbol for a square root is $\sqrt{\quad}$, which is called a **radical sign**.

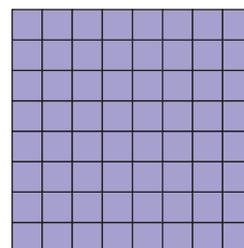


EXAMPLE 2 Finding Square Roots of Perfect Squares

Find each square root.

A $\sqrt{64}$

Method 1: Use a model.



The square root of 64 is 8.

Find each square root.

B $\sqrt{324}$

Method 2: Use a calculator.

Press **2nd** $\sqrt{}$ 324 **ENTER**.

$$\sqrt{324} = 18$$

The square root of 324 is 18.

You can use perfect squares to estimate the square roots of nonperfect squares.

EXAMPLE 3 Estimating Square Roots

Estimate $\sqrt{30}$ to the nearest whole number. Use a calculator to check your answer.

1, 4, 9, 16, 25, 36, . . .

List some perfect squares.

$$25 < 30 < 36$$

Find the perfect squares nearest 30.

$$\sqrt{25} < \sqrt{30} < \sqrt{36}$$

$$5 < \sqrt{30} < 6$$

Find the square roots of 25 and 36.

$$\sqrt{30} \approx 5$$

30 is closer to 25 than to 36.

Check

$$\sqrt{30} \approx 5.477225575$$

Use a calculator to approximate $\sqrt{30}$.

5 is a reasonable estimate.

EXAMPLE 4 Recreation Application

While searching for a lost hiker, a helicopter pilot covers a square area of 150 mi^2 . What is the approximate length of each side of the square area? Round your answer to the nearest mile.



The length of each side of the square is $\sqrt{150}$.

$$144 < 150 < 169$$

Find the perfect squares nearest 150.

$$\sqrt{144} < \sqrt{150} < \sqrt{169}$$

$$12 < \sqrt{150} < 13$$

Find the square roots of 144 and 169.

$$\sqrt{150} \approx 12$$

150 is closer to 144 than to 169.

Each side of the search area is about 12 miles long.

Think and Discuss

1. Explain how to estimate $\sqrt{75}$.
2. Explain how you might find the square root of 3^2 .

GUIDED PRACTICE

See Example 1 Find each square.

1. 4^2

2. 17^2

3. 9^2

4. 15^2

See Example 2 Find each square root.

5. $\sqrt{400}$

6. $\sqrt{9}$

7. $\sqrt{144}$

8. $\sqrt{529}$

See Example 3 Estimate each square root to the nearest whole number. Use a calculator to check your answer.

9. $\sqrt{20}$

10. $\sqrt{45}$

11. $\sqrt{84}$

12. $\sqrt{58}$

See Example 4 13. A Coast Guard ship patrols an area of 125 square miles. The area the ship patrols is a square. About how long is each side of the area? Round your answer to the nearest mile.

INDEPENDENT PRACTICE

See Example 1 Find each square.

14. 3^2

15. 16^2

16. 8^2

17. 11^2

See Example 2 Find each square root.

18. $\sqrt{361}$

19. $\sqrt{16}$

20. $\sqrt{169}$

21. $\sqrt{441}$

See Example 3 Estimate each square root to the nearest whole number. Use a calculator to check your answer.

22. $\sqrt{12}$

23. $\sqrt{39}$

24. $\sqrt{73}$

25. $\sqrt{109}$

See Example 4 26. The area of a square field is 200 ft². What is the approximate length of each side of the field? Round your answer to the nearest foot.**PRACTICE AND PROBLEM SOLVING****Extra Practice**

See page 745.

Estimate each square root to the nearest whole number.

27. $\sqrt{6}$

28. $\sqrt{180}$

29. $\sqrt{145}$

30. $\sqrt{216}$

31. $\sqrt{300}$

32. $\sqrt{420}$

33. $\sqrt{700}$

34. $\sqrt{1,500}$

Use a calculator to find each square root to the nearest tenth.

35. $\sqrt{44}$

36. $\sqrt{253}$

37. $\sqrt{87}$

38. $\sqrt{125}$

39. $\sqrt{380}$

40. $\sqrt{94}$

41. $\sqrt{202}$

42. $\sqrt{571}$

43. **Critical Thinking** An artist is making two square stained-glass windows. One window has a perimeter of 48 inches. The other has an area of 110 square inches. Which window is bigger? Explain.



Earth Science



To find the distance at which an object becomes visible, you can use your distance to the horizon and the object's distance to the horizon.

Given the area, find the missing value for each circle. Use 3.14 for π .

44. $A = 706.9 \text{ m}^2$; $r = \square$

45. $A = 615.44 \text{ yd}^2$; $C = \square$

46. $A = 28.26 \text{ ft}^2$; $d = \square$

47. $A = 3.14 \text{ in}^2$; $r = \square$

Order the numbers from least to greatest.

48. $\sqrt{49}$, $\frac{17}{3}$, 6.5, 8, $\frac{25}{4}$

49. $5\frac{2}{3}$, $\sqrt{25}$, 3^2 , 7.15, $\frac{29}{4}$

50. Find the perimeter of a square whose area is 49 square inches.

51. Earth Science The formula $D = 3.56 \cdot \sqrt{A}$ gives the distance D in kilometers to the horizon from an airplane flying at an altitude A in meters. If a pilot is flying at an altitude of 1,800 m, about how far away is the horizon? Round your answer to the nearest kilometer.

52. Multi-Step For his new room, Darien's grandmother gave him a handmade quilt. The quilt is made up of 16 squares set in 4 rows of 4. The area of each square is 324 in^2 . What are the dimensions of the quilt in inches?



53. Choose a Strategy The figure shows how two squares can be formed by drawing only seven lines. Show how two squares can be formed by drawing only six lines.



54. Write About It Explain the difference between finding the square of a number and finding the square root of a number. Use models and numbers in your explanation.



55. Challenge Find the value of $\sqrt{5^2 + 12^2}$.



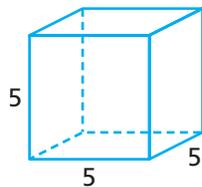
TEST PREP and Spiral Review

56. Multiple Choice Which model represents 5^2 ?

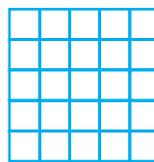
(A)



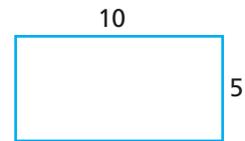
(B)



(C)



(D)



57. Multiple Choice Estimate the value of $\sqrt{87}$ to the nearest whole number.

(F) 9

(G) 10

(H) 11

(J) 12

Classify each triangle according to the lengths of its sides. (Lesson 8-6)

58. 2 in., 3 in., 4 in.

59. 5 cm, 5 cm, 5 cm

60. 8 ft, 6 ft, 8 ft

Given the radius or diameter, find the circumference and area of each circle to the nearest tenth. Use 3.14 for π . (Lesson 9-5)

61. $r = 11 \text{ in.}$

62. $d = 25 \text{ cm}$

63. $r = 3 \text{ ft}$



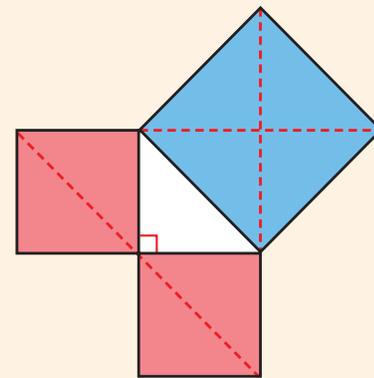
Explore the Pythagorean Theorem



An important and famous relationship in mathematics, known as the Pythagorean Theorem, involves the three sides of a right triangle. Recall that a right triangle is a triangle that has one right angle. If you know the lengths of two sides of a right triangle, you can find the length of the third side.

Activity 1

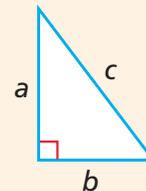
- 1 The drawing at right shows an isosceles right triangle and three squares. Make your own drawing similar to the one shown. (Recall that an isosceles right triangle has two congruent sides and a right angle.)



Cut out the two smaller squares of your drawing, then cut those squares in half along a diagonal. Fit the pieces of the smaller squares on top of the blue square.

Think and Discuss

1. What can you tell about the relationship between the areas of the squares?
- 2a. How does the side length of a square relate to the area of the square?
b. How do the side lengths of the triangle in your drawing relate to the areas of the squares around it?
- c. Write an equation that shows the relationship between the lengths of the sides of the triangle in your drawing. Use the variables a and b to represent the lengths of the two shorter sides of your triangle, and c to represent the length of the longest side.

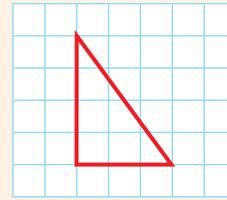


Try This

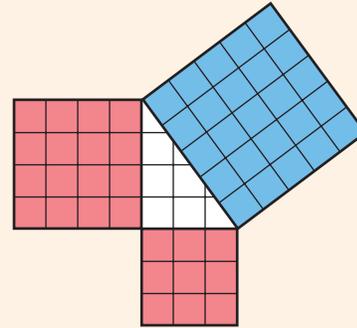
1. Repeat Activity 1 for other isosceles right triangles. Is the relationship that you found true for the areas of the squares around each triangle?

Activity 2

- 1 On graph paper, draw a segment that is 3 units long. At one end of this segment, draw a perpendicular segment that is 4 units long. Draw a third segment to form a triangle. Cut out the triangle.



Cut out a 3-by-3 square and a 4-by-4 square from the same graph paper. Place the edges of the squares against the corresponding sides of the right triangle.



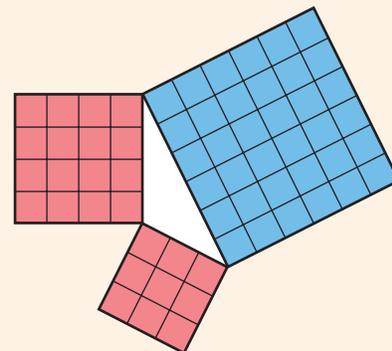
Cut the two squares into individual squares or strips. Arrange the squares into a large square along the third side of the triangle.

Think and Discuss

1. What is the area of each of the three squares? What relationship is there between the areas of the small squares and the area of the large square?
2. What is the length of the third side of the triangle?
3. Substitute the side lengths of your triangle into the equation you wrote in Think and Discuss Problem 2c in Activity 1. What do you find?
4. Do you think the relationship is true for triangles that are not right triangles?

Try This

1. Use graph paper to cut out three squares with sides that are 3 units, 4 units, and 6 units long. Fit the squares together to form a triangle as shown at right. Is the relationship between the areas of the red squares and the area of the blue square the same as the relationship shown in Activity 2? Explain.
2. If you know the lengths of the two short sides of a right triangle are 9 and 12, can you find the length of the longest side? Show your work.
3. If you know the length of the longest side of a right triangle and the length of one of the shorter sides, how would you find the length of the third side?

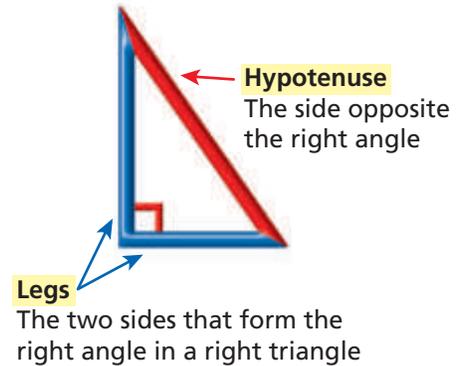


9-8

The Pythagorean Theorem

Learn to use the Pythagorean Theorem to find the length of a side of a right triangle.

One of the first people to recognize the relationship between the sides of a right triangle was the Greek mathematician Pythagoras. This special relationship is called the *Pythagorean Theorem*.



Vocabulary

leg

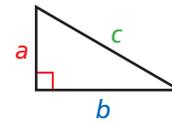
hypotenuse

Pythagorean Theorem

PYTHAGOREAN THEOREM

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

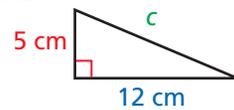


You can use the Pythagorean Theorem to find the length of any side of a right triangle.

EXAMPLE 1 Calculating the Length of a Side of a Right Triangle

Use the Pythagorean Theorem to find each missing measure.

A



$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$25 + 144 = c^2$$

$$169 = c^2$$

$$\sqrt{169} = \sqrt{c^2}$$

$$13 = c$$

The length of the hypotenuse is 13 cm.

Use the Pythagorean Theorem.

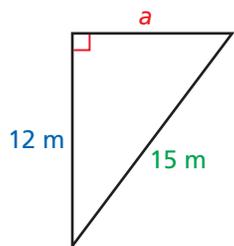
Substitute for a and b .

Evaluate the powers.

Add.

Take the square root of both sides.

B



$$a^2 + b^2 = c^2$$

$$a^2 + 12^2 = 15^2$$

$$a^2 + 144 = 225$$

$$\underline{-144} \quad \underline{-144}$$

$$a^2 = 81$$

$$\sqrt{a^2} = \sqrt{81}$$

$$a = 9$$

The length of the leg is 9 m.

Use the Pythagorean Theorem.

Substitute for b and c .

Evaluate the powers.

Subtract 144 from both sides.

Take the square root of both sides.

EXAMPLE 2 PROBLEM SOLVING APPLICATION



A regulation baseball diamond is a square with sides that measure 90 feet. About how far is it from home plate to second base? Round your answer to the nearest tenth.



1 Understand the Problem

Rewrite the question as a statement.

- Find the distance from home plate to second base.

List the **important information**:

- Drawing a segment between home plate and second base divides the diamond into two right triangles.
- The angle at first base is the right angle, so the segment between home plate and second base is the hypotenuse.
- The base lines are legs, and they are each 90 feet long.

2 Make a Plan

You can use the Pythagorean Theorem to write an equation.

3 Solve

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Use the Pythagorean Theorem.} \\ 90^2 + 90^2 &= c^2 && \text{Substitute for the known variables.} \\ 8,100 + 8,100 &= c^2 && \text{Evaluate the powers.} \\ 16,200 &= c^2 && \text{Add.} \\ 127.279 &\approx c && \text{Take the square root of both sides.} \\ 127.3 &\approx c && \text{Round.} \end{aligned}$$

The distance from home plate to second base is about 127.3 ft.

4 Look Back

The hypotenuse is the longest side of a right triangle. Since the distance from home plate to second base is greater than the distance between the bases, the answer is reasonable.

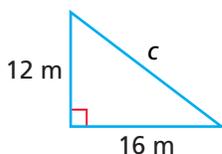
Think and Discuss

1. **Explain** whether it is ever possible to use the Pythagorean Theorem to find an unknown side length of a scalene triangle.
2. **Demonstrate** whether a leg of a right triangle can be longer than the hypotenuse.

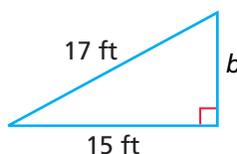
GUIDED PRACTICE

See Example 1 Use the Pythagorean Theorem to find each missing measure.

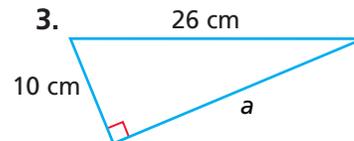
1.



2.

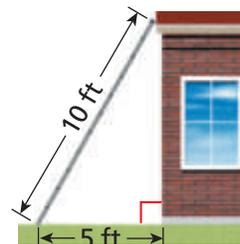


3.



See Example 2

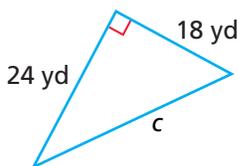
4. A 10 ft ladder is leaning against a wall. If the ladder is 5 ft from the base of the wall, how far above the ground does the ladder touch the wall? Round your answer to the nearest tenth.



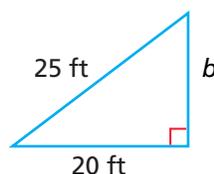
INDEPENDENT PRACTICE

See Example 1 Use the Pythagorean Theorem to find each missing measure.

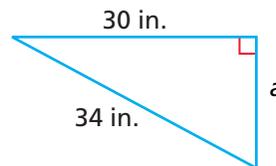
5.



6.

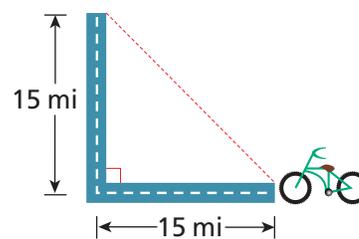


7.



See Example 2

8. James rides his bike 15 miles west. Then he turns north and rides another 15 miles before he stops to rest. How far is James from his starting point when he stops to rest? Round your answer to the nearest tenth.



PRACTICE AND PROBLEM SOLVING

Extra Practice

See page 745.

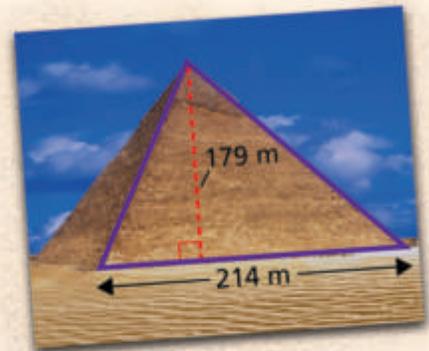
The lengths of two sides of a right triangle are given. Find the length of the third side to the nearest tenth.

9. legs: 5 ft and 8 ft
10. leg: 10 mm; hypotenuse: 15 mm
11. leg: 19 m; hypotenuse: 31 m
12. legs: 21 yd and 20 yd
13. legs: 13.5 in. and 18 in.
14. leg: 13 cm; hypotenuse: 18 cm
15. **Critical Thinking** The numbers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$. When you double each of these values, does the resulting set of numbers also form a Pythagorean triple? Explain.

16. Ancient Egyptians built pyramids to serve as tombs for their kings. One pyramid, called Menkaure, has a square base with an area of about $12,100 \text{ m}^2$.

- a. What is the length of each side of the base?
- b. What is the length of a diagonal of the base?
Round your answer to the nearest tenth.

17. The photograph shows the Pyramid of Khafre in Egypt. Each side of its square base is about 214 meters long. Each triangular side is an isosceles triangle with a height of about 179 meters. What is the area of one side of the pyramid?



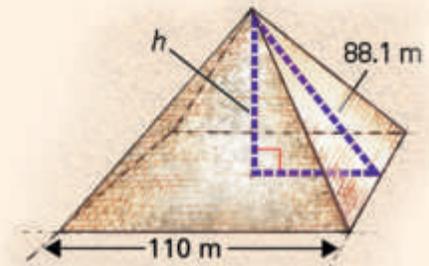
18. Use the Pythagorean Theorem to find the distance from one corner of the Pyramid of Khafre to its peak. Round your answer to the nearest tenth.

19. **Multi-Step** The pyramids were constructed using a unit of measurement called a cubit. There are about 21 inches in 1 cubit. If the height of a pyramid is 471 feet, what is its height in cubits?

20. **Write About It** Given a right triangle, explain how you know which values to substitute into the equation $a^2 + b^2 = c^2$.

21. **Challenge** The pyramid at right has a square base. Find the height of the pyramid to the nearest tenth.

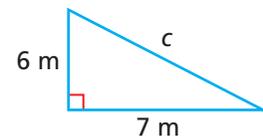
go.hrw.com
Web Extra!
KEYWORD: MS7 Egypt



TEST PREP and Spiral Review

22. **Multiple Choice** Find the missing measure to the nearest tenth.

- (A) 3.6 m (C) 11.8 m
- (B) 9.2 m (D) 85 m



23. **Gridded Response** A 10-foot ladder is leaning against a wall. The bottom of the ladder is 2 feet away from the bottom of the wall. To the nearest tenth, how many feet up the wall will the ladder reach?

Find the measure of the angle formed by the hour and minute hands of a clock at each time. (Lesson 8-4)

- 24. 6:00 25. 3:00 26. 5:00 27. 2:00

Estimate each square root to the nearest whole number. (Lesson 9-7)

- 28. $\sqrt{140}$ 29. $\sqrt{60}$ 30. $\sqrt{200}$ 31. $\sqrt{30}$

Quiz for Lessons 9-7 Through 9-8

 **9-7 Squares and Square Roots**

Find each square.

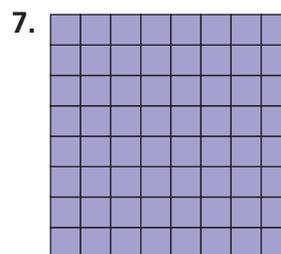
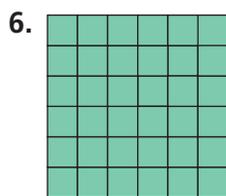
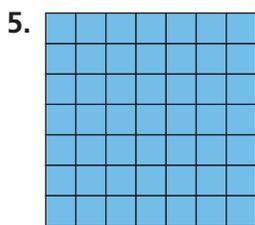
1. 21^2

2. 7^2

3. 12^2

4. 13^2

Name the square and the square root represented by each model.



Find each square root.

8. $\sqrt{841}$

9. $\sqrt{1,089}$

10. $\sqrt{81}$

11. $\sqrt{576}$

Estimate each square root to the nearest whole number.

Use a calculator to check your answer.

12. $\sqrt{40}$

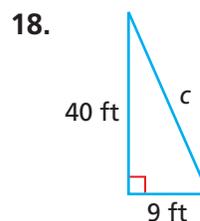
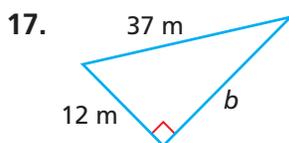
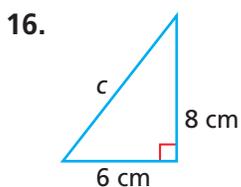
13. $\sqrt{85}$

14. $\sqrt{12}$

15. $\sqrt{33}$

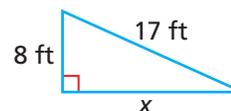
 **9-8 The Pythagorean Theorem**

Use the Pythagorean Theorem to find each missing measure.



19. Thomas likes to jog at Memorial Park. The running trail he follows is in the shape of a right triangle. He knows one leg of the path is 1.8 miles, and the other leg is 3.2 miles. What is the distance of the third side of the trail to the nearest tenth of a mile?

20. Audrey built a ramp for the set of the new musical at her school. The height of the ramp is 8 feet, and the hypotenuse is 17 feet. What is the length of the ramp's base?



The lengths of two sides of a right triangle are given. Find the length of the third side to the nearest tenth.

21. leg: 14.3 m; hypotenuse: 22 m

22. legs: 10 yd and 24 yd

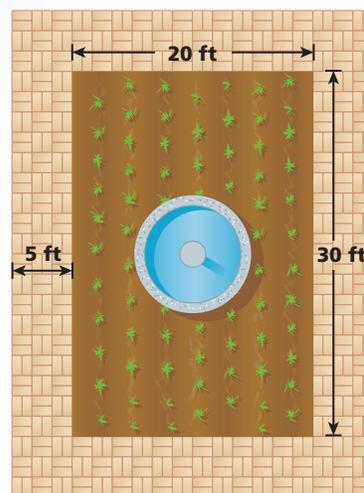
23. legs: 12.4 in. and 9.0 in.

24. leg: 2.5 cm; hypotenuse: 8 cm



Gabriella's Garden Gabriella is designing a rectangular garden for her school. As shown in the figure, the garden will be surrounded by a walkway that is 5 feet wide.

1. Gabriella wants to put a fence around the outside of the walkway. How much fencing does she need?
2. Gabriella is going to plant cabbage seedlings around the inside edge of the walkway. The seedlings should be planted 12 inches apart. How many seedlings will she need?
3. The design calls for a circular fountain in the center of the garden. The fountain's diameter will be 10 feet. To the nearest tenth of a foot, what will be the length of the concrete border that forms the edge of the fountain?
4. What is the area of the remaining land that is available for plants? Explain.
5. To help protect and enrich the soil, Gabriella plans to cover the planted part of the garden with mulch. One bag of mulch covers 18 square feet. How many bags should she buy?
6. To celebrate the opening of the garden, Gabriella wants to hang streamers from poles at the outer corners of the walkway, creating an X. How many feet of streamers will Gabriella need?



EXTENSION

Identifying and Graphing Irrational Numbers

Learn to classify numbers as rational or irrational and graph them on a number line.

Vocabulary

irrational numbers

Recall from Lesson 2-11 that a rational number can be written as a fraction with integers for its numerator and denominator. When rational numbers are written in decimal form, the decimal may be terminating or nonterminating. If a rational number is nonterminating, then it has a repeating pattern.

A decimal that is nonterminating with no repeating pattern is an **irrational number**.

Rational		Irrational
Terminating	Nonterminating, Repeating	Nonterminating, Nonrepeating
$\frac{1}{8} = 0.125$	$\frac{1}{3} = 0.333\dots$, or $0.\bar{3}$	$\sqrt{2} = 1.414213562\dots$
$\sqrt{9} = 3$	$\frac{2}{11} = 0.181818\dots$, or $0.1\bar{8}$	$\pi = 3.1415926\dots$

EXAMPLE 1 Identifying Rational and Irrational Numbers

Identify each number as rational or irrational. Justify your answer.

A $\frac{2}{5}$

$$\frac{2}{5} = 0.4$$

Write the number in decimal form.

Because its decimal form is terminating, $\frac{2}{5}$ is rational.

B $\frac{5}{6}$

$$\frac{5}{6} = 0.8333\dots$$
, or $0.8\bar{3}$

Write the number in decimal form.

Because its decimal form is nonterminating and repeating, $\frac{5}{6}$ is rational.

C $\sqrt{16}$

$$\sqrt{16} = 4$$

Write the number in decimal form.

Because its decimal form is terminating, $\sqrt{16}$ is rational.

D $\sqrt{7}$

$$\sqrt{7} = 2.645751311\dots$$

Write the number in decimal form.

There is no pattern in the decimal form of $\sqrt{7}$. It is a nonterminating, nonrepeating decimal. So $\sqrt{7}$ is irrational.

Remember!

By definition, any ratio of integers is a rational number.

Every point on the number line corresponds to a real number, either a rational number or an irrational number. Between every two real numbers there is always another real number.

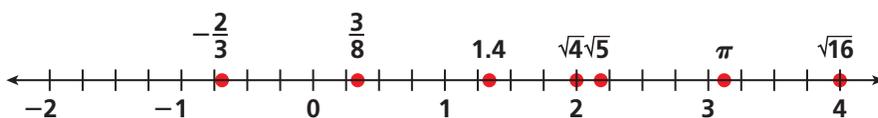
EXAMPLE 2 Graphing Rational and Irrational Numbers

Graph the list of numbers on a number line. Then order the numbers from least to greatest.

$$1.4, \sqrt{5}, \frac{3}{8}, \pi, -\frac{2}{3}, \sqrt{4}, \sqrt{16}$$

Write all the numbers in decimal form, and then graph them.

$$1.4, \sqrt{5} \approx 2.236, \frac{3}{8} = 0.375, \pi \approx 3.14, -\frac{2}{3} = -0.\bar{6}, \sqrt{4} = 2.0, \sqrt{16} = 4.0$$



From left to right on the number line, the numbers appear from least to greatest: $-\frac{2}{3} < \frac{3}{8} < 1.4 < \sqrt{4} < \sqrt{5} < \pi < \sqrt{16}$.

EXTENSION

Exercises

Identify each number as rational or irrational. Justify your answer.

- | | | | |
|-------------------|--------------------|---------------------|------------------|
| 1. $\sqrt{8}$ | 2. $\frac{5}{11}$ | 3. $\frac{7}{8}$ | 4. $\sqrt{36}$ |
| 5. $\frac{3}{13}$ | 6. $\sqrt{14}$ | 7. 2.800 | 8. $\frac{5}{6}$ |
| 9. $\sqrt{5}$ | 10. $\frac{6}{24}$ | 11. $\frac{10}{33}$ | 12. $\sqrt{18}$ |

Graph each list of numbers on a number line. Then order the numbers from least to greatest.

- | | |
|---|---|
| 13. 2.6, 0.5, $\sqrt{3}$, $-\frac{7}{10}$, $\frac{1}{3}$ | 14. $\sqrt{12}$, $\frac{3}{8}$, -0.65 , $\frac{5}{9}$, $\sqrt{11}$ |
| 15. -1.3 , $\sqrt{15}$, 3.1, $-\frac{2}{5}$, $\sqrt{4}$ | 16. -2.1 , $-\frac{9}{10}$, $\sqrt{1}$, -1.5 , $\sqrt{9}$ |

Name the two perfect squares that each square root lies between. Then graph the square root on a number line, and justify its placement.

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 17. $\sqrt{34}$ | 18. $\sqrt{46}$ | 19. $\sqrt{14}$ | 20. $\sqrt{6}$ |
| 21. $\sqrt{99}$ | 22. $\sqrt{63}$ | 23. $\sqrt{71}$ | 24. $\sqrt{13}$ |

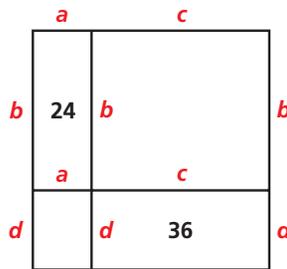
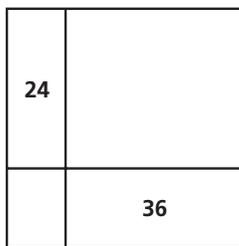
25. **What's the Error?** A classmate tells you that the square root of any number is irrational. Explain why the classmate is incorrect.

Game Time

Shape Up

Rectangles

The square below has been divided into four rectangles. The areas of two of the rectangles are given. If the length of each of the segments in the diagram is an integer, what is the area of the original square?

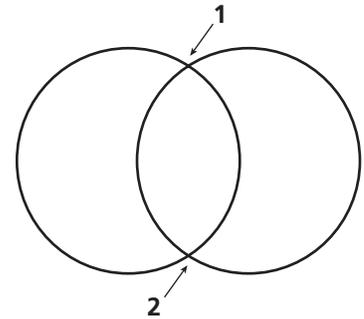


(Hint: Remember $a + c = b + d$.)

Use different lengths and a different answer to create your own version of this puzzle.

Circles

What is the maximum number of times that six circles of the same size can intersect? To find the answer, start by drawing two circles that are the same size. What is the greatest number of times they can intersect? Add another circle, and another, and so on.



Circles and Squares

Two players start with a sequence of circles and squares. Before beginning the game, each player chooses whether to be a "circle" or a "square." The goal of the game is to have the final remaining shape be the shape you chose to be. Shapes are removed from the sequence according to the following rules: On each move, a player selects two shapes. If the shapes are identical, they are replaced with one square. If the shapes are different, they are replaced with one circle.

A complete copy of the rules and game pieces are available online.



go.hrw.com
Game Time Extra
 KEYWORD: MS7 Games



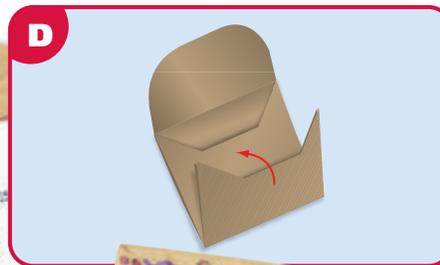
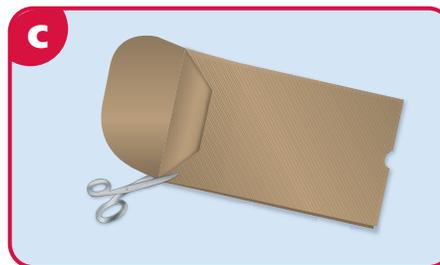
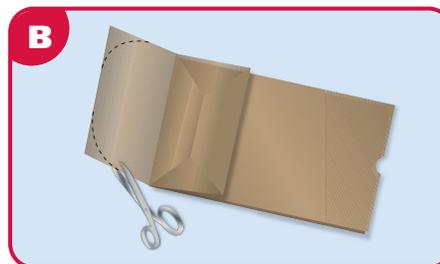
Materials

- lunch bag
- scissors
- tape
- markers
- index cards

PROJECT Bag o' Measurement

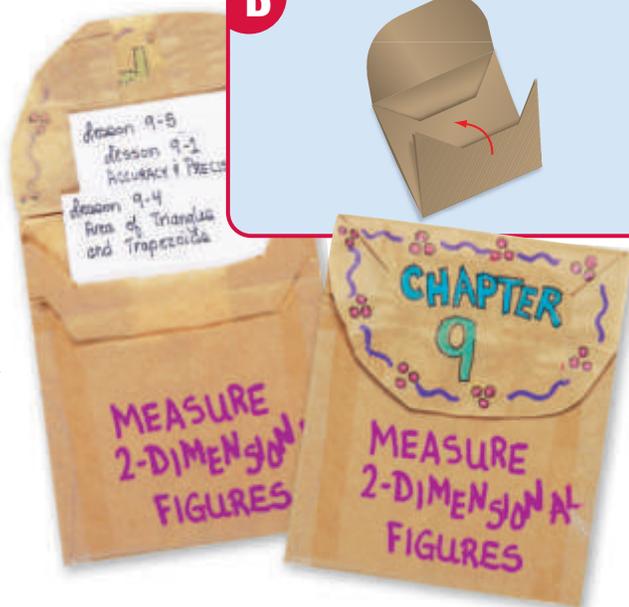
This bag of index cards will help you organize your notes on measuring two-dimensional figures.

- 1 Hold the lunch bag with the flap facing you at the top. Cut a thin strip from the flap as shown. **Figure A**
- 2 Cut along the sides of the flap so you can open it up. Then use your scissors to round off the corners at the top of the flap. **Figure B**
- 3 Fold up the bottom part of the flap. Then trim this part of the flap by cutting out a trapezoid as shown. **Figure C**
- 4 Cut another trapezoid from the bottom edge of the bag by cutting through all the layers. Then fold up the bottom of the bag to make two pockets, one below the other. **Figure D**
- 5 Tape the sides of the bag together to close the pockets. Fold down the flap and label it with the number and title of the chapter.



Taking Note of the Math

Use index cards to record measurement formulas, the Pythagorean Theorem, and other important facts from the chapter. Store the cards in the pockets of the bag.



Vocabulary

accuracy	518	leg	556	Pythagorean Theorem	556
area	530	perfect square	550	radical sign	550
circumference	525	perimeter	524	significant digits	518
hypotenuse	556	precision	518	square root	550

Complete the sentences below with vocabulary words from the list above.

- The longest side of a right triangle is called the ___?___.
- The ___?___ is the distance around a circle.
- ___?___ is the level of detail an instrument can measure.
- A(n) ___?___ is one of the two equal factors of a number.

9-1 Accuracy and Precision (pp. 518–521)**EXAMPLE**

- Determine the number of significant digits in 705.4 mL.

The digits 7, 5, and 4 are nonzero digits, and 0 is between two nonzero digits. So 705.4 mL has 4 significant digits.

EXERCISES

Determine the number of significant digits in each measurement.

- 0.450 kg
- 30,000 lb
- 900.5 cm
- 6,703.0 ft
- 0.00078 g
- 1,204 gal

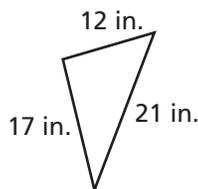
9-2 Perimeter and Circumference (pp. 524–527)**EXAMPLE**

- Find the perimeter of the triangle.

$$P = 12 + 17 + 21$$

$$P = 50$$

The perimeter is 50 in.



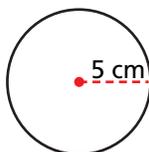
- Find the circumference of the circle. Use 3.14 for π .

$$C = 2\pi r$$

$$C \approx 2 \cdot 3.14 \cdot 5$$

$$C \approx 31.4$$

The circumference is about 31.4 cm.

**EXERCISES**

Find the perimeter of each polygon.

-
-

Find the circumference of each circle to the nearest tenth. Use 3.14 for π .

-
-

9-3 Area of Parallelograms (pp. 530–533)

EXAMPLE

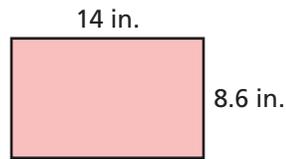
- Find the area of the rectangle.

$$A = \ell w$$

$$A = 14 \cdot 8.6$$

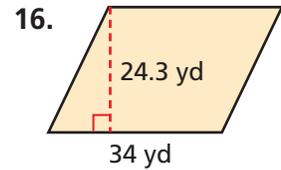
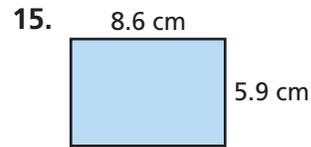
$$A = 120.4$$

The area of the rectangle is 120.4 in^2 .



EXERCISES

Find the area of each polygon.



17. Rose is drawing a portrait for her art class. She is using a sheet of paper that is 6 inches wide and 12 inches long. What is the area of the art paper in square inches?

9-4 Area of Triangles and Trapezoids (pp. 534–537)

EXAMPLE

- Find the area of the triangle.

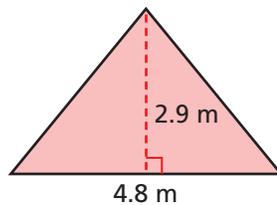
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(4.8 \cdot 2.9)$$

$$A = \frac{1}{2}(13.92)$$

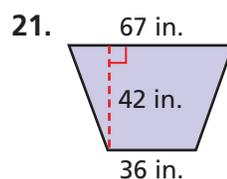
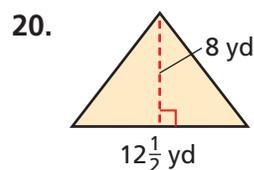
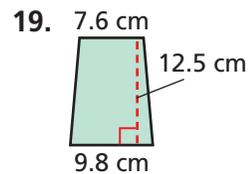
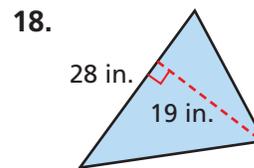
$$A = 6.96$$

The area of the triangle is 6.96 m^2 .



EXERCISES

Find the area of each polygon.



9-5 Area of Circles (pp. 538–541)

EXAMPLE

- Find the area of the circle to the nearest tenth. Use 3.14 for π .

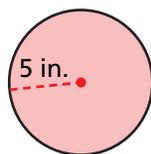
$$A = \pi r^2$$

$$A \approx 3.14 \cdot 5^2$$

$$A \approx 3.14 \cdot 25$$

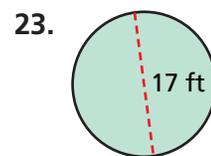
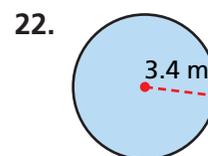
$$A \approx 78.5$$

The area of the circle is about 78.5 in^2 .



EXERCISES

Find the area of each circle to the nearest tenth. Use 3.14 for π .



24. The minute hand on a clock is 9 inches long. What is the area of the circle the minute hand covers after one hour? Give your answer in square inches.

9-6 Area of Irregular Figures (pp. 542–545)

EXAMPLE

- Find the area of the irregular figure.

Separate the figure into a rectangle and a triangle.

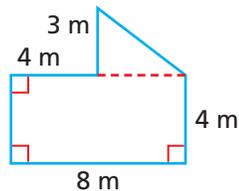
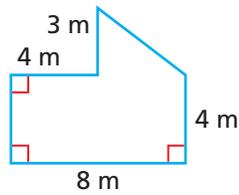
$$A = \ell w$$

$$= 4 \cdot 8 = 32 \text{ m}^2$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(3 \cdot 4) = 6 \text{ m}^2$$

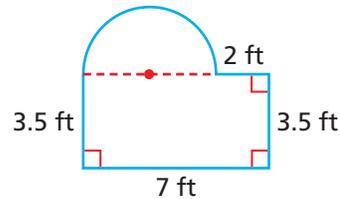
$$A = 32 + 6 = 38 \text{ m}^2$$



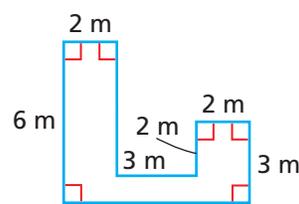
EXERCISES

Find the area of each figure. Use 3.14 for π .

25.



26.



9-7 Squares and Square Roots (pp. 550–553)

EXAMPLE

- Estimate $\sqrt{71}$ to the nearest whole number.

$$64 < 71 < 81 \quad \textit{Find the perfect squares nearest 71.}$$

$$\sqrt{64} < \sqrt{71} < \sqrt{81}$$

$$8 < \sqrt{71} < 9$$

Find the square roots of 64 and 81.

Since 71 is closer to 64 than to 81, $\sqrt{71} \approx 8$.

EXERCISES

Estimate each square root to the nearest whole number.

27. $\sqrt{29}$

28. $\sqrt{92}$

29. $\sqrt{106}$

30. $\sqrt{150}$

31. The area of Rita's square vegetable garden is 265 ft^2 . What is the length of each side of the garden to the nearest foot?

9-8 Pythagorean Theorem (pp. 556–559)

EXAMPLE

- Use the Pythagorean Theorem to find the missing measure.

$$a^2 + b^2 = c^2$$

$$9^2 + 12^2 = c^2$$

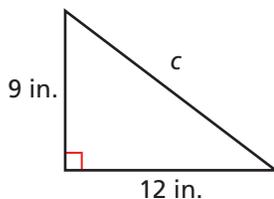
$$81 + 144 = c^2$$

$$225 = c^2$$

$$\sqrt{225} = \sqrt{c^2}$$

$$15 = c$$

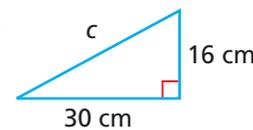
The hypotenuse is 15 in.



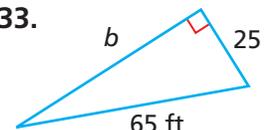
EXERCISES

Use the Pythagorean Theorem to find each missing measure.

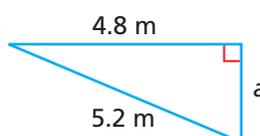
32.



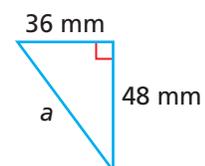
33.



34.



35.



Choose the more precise measurement in each pair.

1. 80 m, 7.9 cm 2. 18 yd, 5 mi 3. 500 lb, 18 oz

Calculate. Use the correct number of significant digits in each answer.

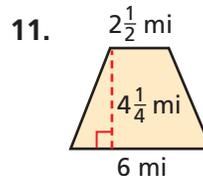
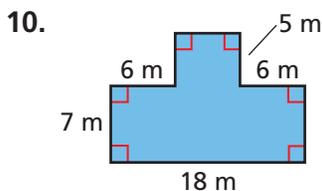
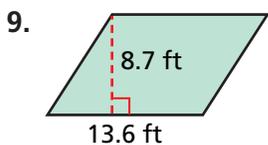
4. $5.6 \text{ lb} \div 2.59$ 5. $3.14 \cdot 125 \text{ cm}$ 6. $5.882 \text{ in.} + 5.17 \text{ in.}$

7. Find the perimeter of the trapezoid.



8. The opening of a playscape tunnel has a circumference of 25 ft. What is the radius of the tunnel to the nearest tenth?

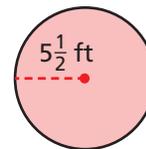
Find the area of each figure.



12. The area of a rectangular computer lab is 660 ft^2 , and the width is 22 ft. What is the length of the computer lab?
13. The area of a circular fountain is 66 cm^2 . What is its radius to the nearest tenth?

Use the diagram for Items 14 and 15.

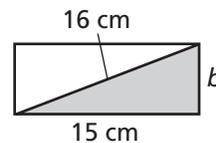
14. Find the circumference of the circle to the nearest tenth.
15. Find the area of the circle to the nearest tenth.



Find each square or square root.

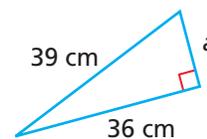
16. 15^2 17. 23^2 18. $\sqrt{1,600}$ 19. $\sqrt{961}$

20. The tiles of Sara's new floor are black and white as shown. What is the missing length to the nearest tenth?
21. Triangle Park has a trail that follows the path of a right triangle. One leg of the trail is 2.1 miles, and the other leg is 3.0 miles. What is the distance of the third side of the trail to the nearest tenth of a mile?



Use the diagram at right for Items 22 and 23.

22. Use the Pythagorean Theorem to find the missing measure.
23. Find the area of the triangle.





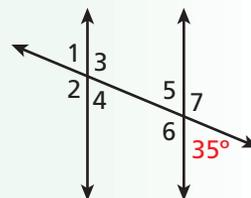
Multiple Choice: Context-Based Test Items

Sometimes a multiple-choice test item requires you to use information in the answer choices to determine which choice fits the context of the problem.

EXAMPLE 1

Which statement is supported by the figure?

- (A) $\angle 1$ and $\angle 4$ are supplementary. (C) The measure of $\angle 7$ is 35° .
 (B) $\angle 3$ and $\angle 2$ are vertical angles. (D) $\angle 5$ and $\angle 6$ are congruent.



Read each answer choice to find the best answer.

- Choice A:** $\angle 1$ and $\angle 4$ are vertical angles and therefore, congruent. Congruent angles are supplementary only if they are right angles. $\angle 1$ and $\angle 4$ measure 35° .
- Choice B:** $\angle 3$ and $\angle 2$ are vertical angles. This is the correct answer choice.
- Choice C:** The measure of $\angle 7$ cannot be 35° because $\angle 7$ is supplementary to a 35° angle. Therefore, $\angle 7$ has a measure of 145° .
- Choice D:** $\angle 5$ and $\angle 6$ are supplementary angles but not right angles. Supplementary angles are congruent only if they are both right angles.

EXAMPLE 2

Which two figures have the same area?

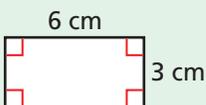


Figure I

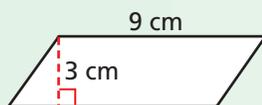


Figure II

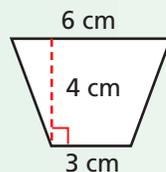


Figure III

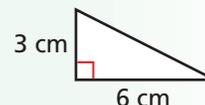


Figure IV

- (F) Figure I and Figure II (H) Figure II and Figure III
 (G) Figure I and Figure III (J) Figure I and Figure IV

Find the areas of all four figures and compare them.

Figure I: $3 \cdot 6 = 18 \text{ cm}^2$

Figure III: $\frac{1}{2} \cdot 4(3 + 6) = 18 \text{ cm}^2$

Figure II: $3 \cdot 9 = 27 \text{ cm}^2$

Figure IV: $\frac{1}{2} \cdot 6 \cdot 3 = 9 \text{ cm}^2$

Figures I and III have the same area. Choice G is correct.



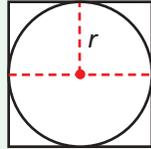
Do not choose an answer until you have read all of the answer choices.

Read each test item and answer the questions that follow.

Item A

The area of the square is 16 cm^2 . Which of the following is NOT correct about the circle?

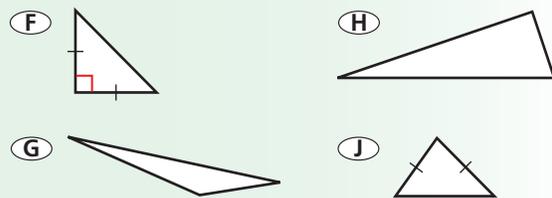
- (A) $C = 4\pi \text{ cm}$
- (B) $A = 16\pi \text{ cm}^2$
- (C) $d = 4 \text{ cm}$
- (D) $r = 2 \text{ cm}$



- Since only one answer choice has incorrect information, why can you automatically eliminate answer choices C and D?
- How can you find the side length of the square? What does the side length tell you about the circle?
- Use your answer to Problem 2 to determine whether answer choice A has correct information.
- How can you tell that choice B is the correct answer?

Item B

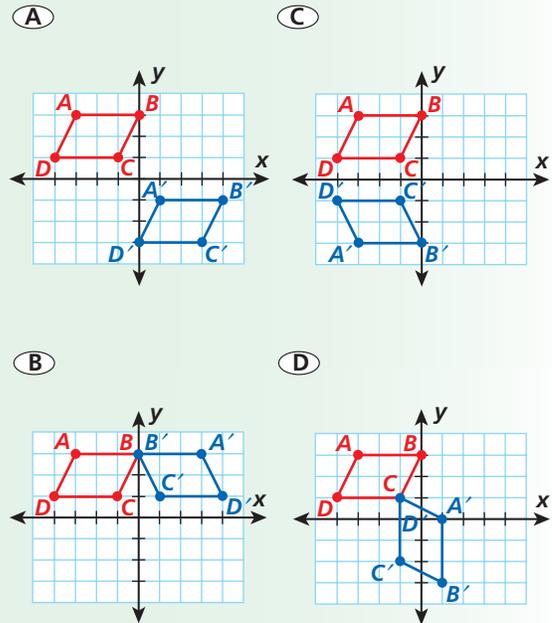
Which figure is an acute isosceles triangle?



- What is an acute triangle?
- What is an isosceles triangle?
- Why is choice F incorrect?

Item C

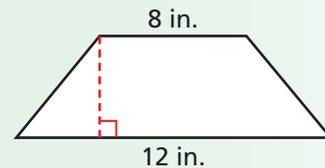
Which graph represents a reflection across the x -axis?



- Which answer choices do NOT show reflections?
- What is a reflection across the x -axis?

Item D

The area of the trapezoid is 30 in^2 . Which equation CANNOT be used to find the height of the trapezoid?



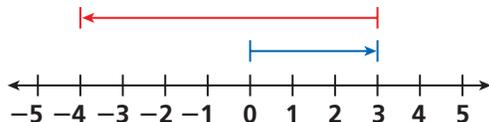
- (F) $30 = \frac{1}{2}(8 + 12)h$
- (G) $60 = (8 + 12)h$
- (H) $30 = \frac{1}{2}(8 - 12)h$
- (J) $\frac{1}{2}(8 + 12)h = 30$

- What is the formula for the area of a trapezoid?
- What steps would you take to solve the formula for h ?

Cumulative Assessment, Chapters 1–9

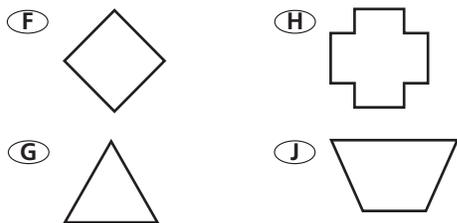
Multiple Choice

1. Which expression is represented by the model below?



- (A) $3 + (-7)$ (C) $-3 + 7$
 (B) $3 + 7$ (D) $-3 + (-7)$

2. Which figure has only one line of symmetry?



3. If $x + 2 = y$ and $y = 4^2$, what is the value of $x + y$?

- (A) 14 (C) 22
 (B) 16 (D) 30

4. You invest \$200 into a simple interest savings account for 5 years and earn \$60 in interest. What interest rate did you earn?

- (F) 1.5% (H) 30%
 (G) 6% (J) 33.3%

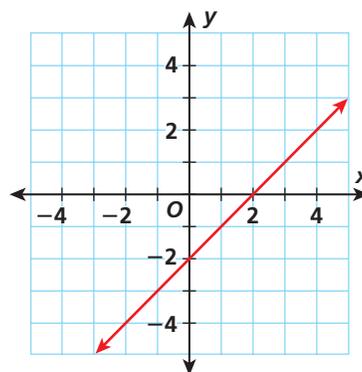
5. A color printer is designed to print 8 pages per minute. How many pages can the printer print in 13 minutes?

- (A) 1.6 pages (C) 84 pages
 (B) 21 pages (D) 104 pages

6. For which radius r is the area of a circle equal to 153.86 square inches?

- (F) $r = 7$ in. (H) $r = 24.5$ in.
 (G) $r = 12.5$ in. (J) $r = 49$ in.

7. Which equation describes the graph?

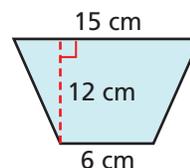


- (A) $y = x - 2$ (C) $y = 2x + 1$
 (B) $y = x + 2$ (D) $y = 2x - 2$

8. Seventy percent of historical figures pictured on U.S. currency do not have facial hair. What is the decimal equivalent of this value?

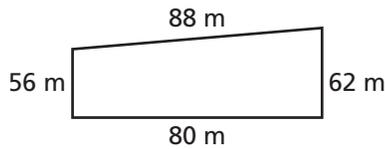
- (F) 0.07 (H) 7.0
 (G) 0.70 (J) 70

9. What is the area of the trapezoid?



- (A) 81 cm^2 (C) 135 cm^2
 (B) 126 cm^2 (D) 252 cm^2

10. Paul plans to build a fence around the perimeter of his property. How much fencing does he need?



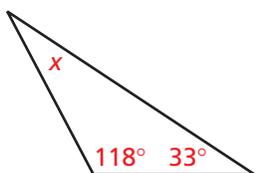
- (F) 286 m (H) 4,480 m
 (G) 294 m (J) 5,456 m
11. Gretchen bought six muffins for \$3.19, four bottles of juice at \$1.25 each, and a bag of apples at \$0.89 per pound. She gave the cashier \$20. What other information is necessary to find Gretchen's correct change?
- (A) Cost of one muffin
 (B) Total cost of the juice
 (C) Number of pounds of apples bought
 (D) Reason for buying the food



Write your explanations to short- and extended-response questions as complete sentences.

Gridded Response

12. How many significant digits are in the measurement 0.00410 milligrams?
13. The diameter of a CD is about 12 cm. What is the circumference of the CD to the nearest tenth of a centimeter? Use 3.14 for π .
14. What is the x -coordinate of the point $(-2, 6)$ after it is translated 5 units right and 7 units down?
15. What is the measure in degrees of $\angle x$ in the triangle below?

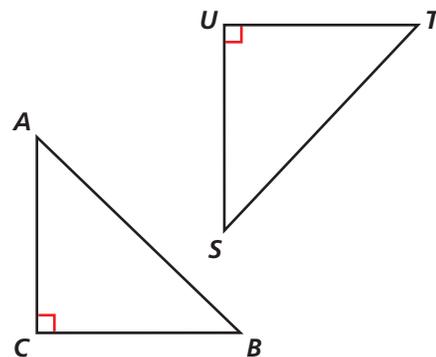


Short Response

16. The tennis team had a pizza party at the end of the season. The 17 team members spent a total of \$51.95 on pizza and \$6.70 on drinks. What is the average amount each team member spent for the party? Show your work.
17. Laurie wants to paste a circular photo onto a rectangular piece of cardboard. The area of the photo is 50.24 in^2 . What are the smallest possible dimensions the piece of cardboard can have and still hold the entire photo? Use 3.14 for π and explain your answer.
18. Find the perimeter and area of a rectangle with length 12 m and width 7 m. Then find the side length of a square that has the same area as the rectangle. Round your answers to the nearest meter, and show your work.

Extended Response

19. Use $\triangle ABC$ and $\triangle STU$ for the following problems.



- a. If $AB = 17 \text{ m}$ and $AC = 8 \text{ m}$, what theorem can you use to find CB ? Find CB , and show your work.
- b. If $ST = 10 \text{ m}$ and $\triangle ABC$ is similar to $\triangle STU$, what ratio can you use to find SU and UT ? Show how to find SU and UT to the nearest tenth of a meter.
- c. Find the difference in the areas of the two triangles.