

# Measurement: Three-Dimensional Figures

## 10A Volume

**LAB** Sketch Three-Dimensional Figures from Different Views

**10-1** Introduction to Three-Dimensional Figures

**LAB** Explore the Volume of Prisms and Cylinders

**10-2** Volume of Prisms and Cylinders

**10-3** Volume of Pyramids and Cones

## 10B Surface Area

**LAB** Use Nets to Build Prisms and Cylinders

**10-4** Surface Area of Prisms and Cylinders

**LAB** Investigate the Surface Areas of Similar Prisms

**10-5** Changing Dimensions

**LAB** Explore Changes in Dimensions

**Ext** Cross Sections

**MULTI-STEP TEST PREP**

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**Chapter Project Online**

KEYWORD: MS7 Ch10

Pyramid	Location	Height (m)	Base Length (m)
El Castillo	Chichén Itzá, Mexico	55.5	79.0
Tikal	Tikal, Guatemala	30.0	80.0
Pyramid of the Sun	Teotihuacán, Mexico	63.0	225.0

## Career *Archaeological Architect*

Did you ever wonder how the pyramids were built? Archaeologists who are also architects combine a love of the past with the skills of a building designer to study the construction of ancient buildings.

In recent years, archaeological architects have built machines like those used in ancient times to demonstrate how the pyramids might have been constructed. The table shows the dimensions of a few famous pyramids.



# ARE YOU READY?

## Vocabulary

Choose the best term from the list to complete each sentence.

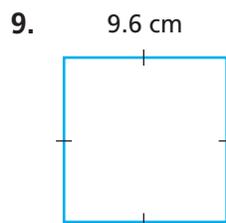
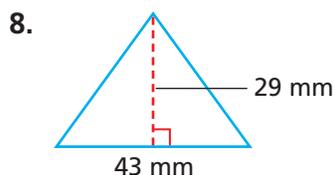
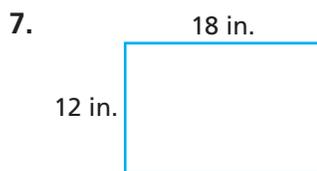
1. A polygon with six sides is called a(n)    ?.
2.    ? figures are the same size and shape.
3. A(n)    ? is a ratio that relates the dimensions of two similar objects.
4. The formula for the    ? of a circle can be written as  $\pi d$  or  $2\pi r$ .
5.    ? figures are the same shape but not necessarily the same size.
6. A polygon with five sides is called a(n)    ?.

area  
circumference  
congruent  
hexagon  
pentagon  
scale factor  
similar

Complete these exercises to review skills you will need for this chapter.

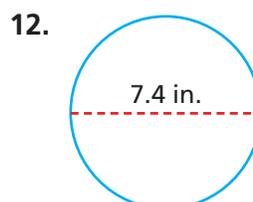
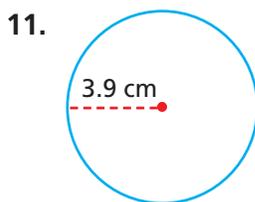
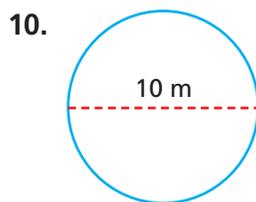
## Area of Squares, Rectangles, Triangles

Find the area of each figure.



## Area of Circles

Find the area of each circle to the nearest tenth. Use 3.14 for  $\pi$ .



## Find the Cube of a Number

Find each value.

13.  $3^3$

14.  $8^3$

15.  $2.5^3$

16.  $6.2^3$

17.  $10^3$

18.  $5.9^3$

19.  $800^3$

20.  $98^3$

### Where You've Been

#### Previously, you

- found the area of polygons and irregular figures.
- compared the relationship between a figure's perimeter and its area.

### In This Chapter

#### You will study

- finding the volume of prisms, cylinders, pyramids, and cones.
- using nets to find the surface area of prisms and cylinders.
- finding the volume and surface area of similar three-dimensional figures.

### Where You're Going

#### You can use the skills learned in this chapter

- to determine the amount of materials needed to build a doghouse.
- to convert dimensions of a model to real-world dimensions.

### Key Vocabulary/Vocabulario

base	base de una figura tridimensional
edge	arista
face	cara
net	plantilla
polyhedron	poliedro
prism	prisma
surface area	área total
vertex of a polyhedron	vértice de un poliedro
volume	volumen

### Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. Note the Spanish translation of *surface area* in the table above. What does the term *área total* tell you about the meaning of **surface area**?
2. The word *edge* comes from the Latin word *acer*, meaning “sharp.” How does the Latin root help you define an **edge** of a three-dimensional figure?
3. The word *vertex* can mean “peak” or “highest point.” What part of a cone or pyramid is the **vertex**?
4. The word *prism* comes from the Greek word *priein*, meaning “to saw.” How might you describe a **prism** in terms of something sawn or cut off?

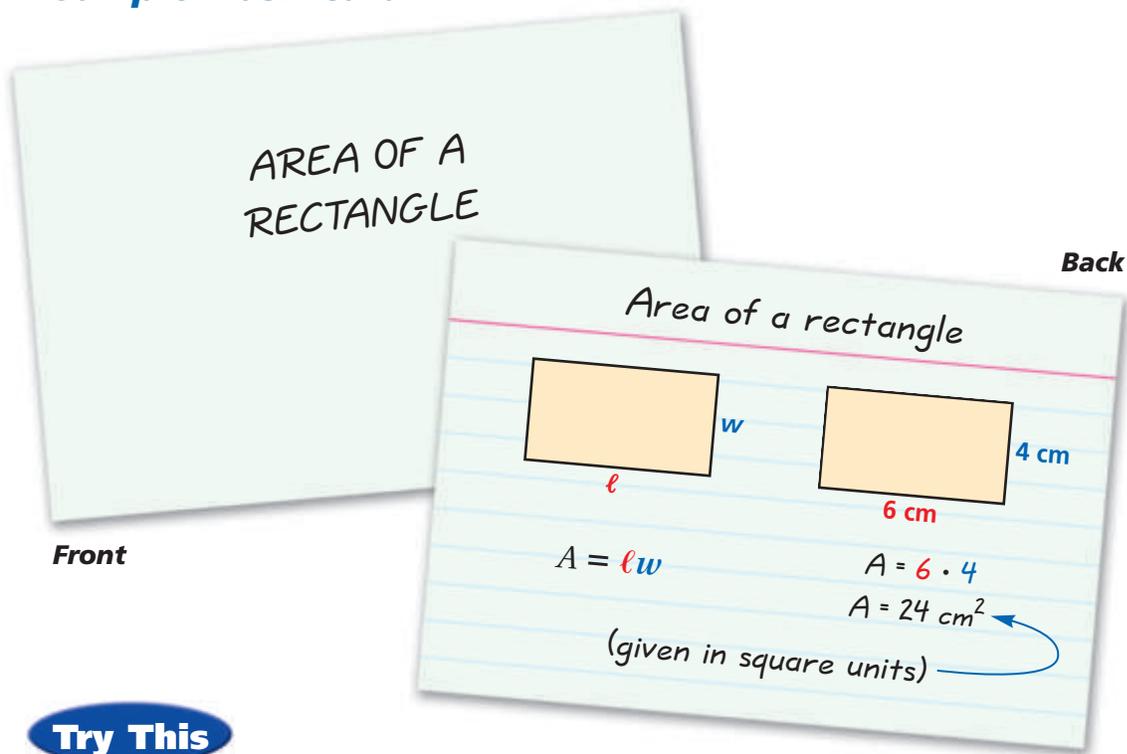
### Study Strategy: Learn and Use Formulas

Throughout this chapter, you will be introduced to many formulas. Although memorizing these formulas is helpful, understanding the concepts on which they are based will help you to re-create the formula if you happen to forget.

One way to memorize a formula is to use flash cards. Write the formula on an index card and review it often. Include a diagram and an example. Add any notes that you choose, such as when to use the formula.

In Lesson 9-3, you learned the formula for area of a rectangle.

### Sample Flash Card



### Try This

1. Create flash cards for some of the formulas from the previous chapters.
2. Describe a plan to help you memorize the formulas in Chapters 9 and 10.



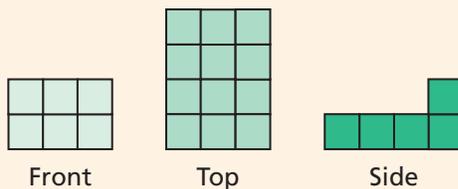
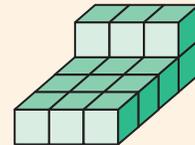
# Sketch Three-Dimensional Figures from Different Views



Three-dimensional figures often look different from different points of view. You can use centimeter cubes to help you visualize and sketch three-dimensional figures.

## Activity 1

- 1 Use centimeter cubes to build the three-dimensional figure at right.
- 2 Now view the figure from the front and draw what you see. Then view the figure from the top and draw what you see. Finally, view the figure from the side and draw what you see.

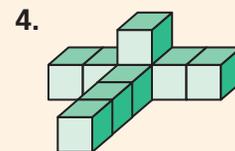
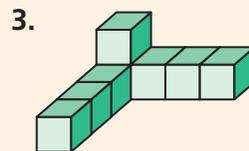
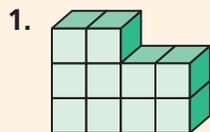


## Think and Discuss

1. How many cubes did you use to build the three-dimensional figure?
2. How could you add a cube to the figure without changing the top view?
3. How could you remove a cube from the figure without changing the side view?

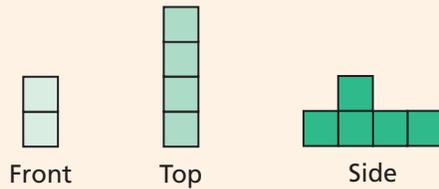
## Try This

Use centimeter cubes to build each three-dimensional figure. Then sketch the front, top, and side views.



## Activity 2

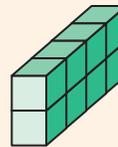
- Use centimeter cubes to build a figure that has the front, top, and side views shown.



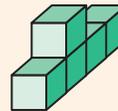
- You can build the figure by first making a simple figure that matches the front view.



- Now add cubes so that the figure matches the top view.



- Finally, remove cubes so that the figure matches the side view. Check that the front and top views are still correct for the figure that you built.



## Think and Discuss

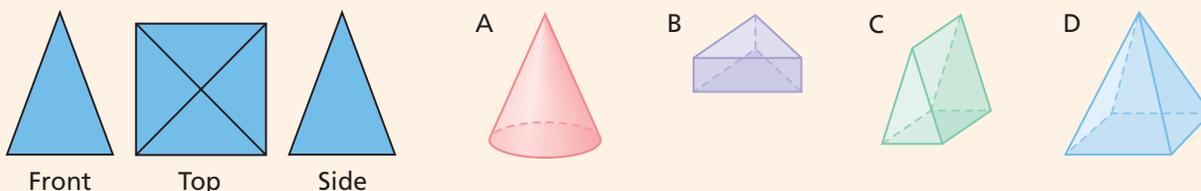
- Discuss whether there is another step-by-step method for building the above figure. If so, is the final result the same?

## Try This

The front, top, and side views of a figure are shown. Use centimeter cubes to build the figure. Then sketch the figure.

- 
- 

- The views below represent a three-dimensional figure that cannot be built from cubes. Determine which three-dimensional figure matches the views.



# 10-1

## Introduction to Three-Dimensional Figures

**Learn** to identify various three-dimensional figures.

### Vocabulary

face

edge

polyhedron

vertex

base

prism

pyramid

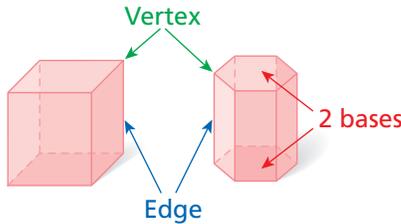
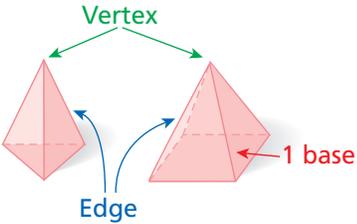
cylinder

cone

Three-dimensional figures have three dimensions: length, width, and height. A flat surface of a three-dimensional figure is a **face**. An **edge** is where two faces meet.

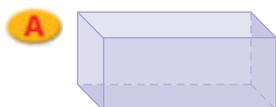
A **polyhedron** is a three-dimensional figure whose faces are all polygons. A **vertex** of a polyhedron is a point where three or more edges meet. The face that is used to name a polyhedron is called a **base**.

A *prism* has two bases, and a *pyramid* has one base.

Prisms	Pyramids
<p>A <b>prism</b> is a polyhedron that has two parallel, congruent bases. The bases can be any polygon. The other faces are parallelograms.</p> 	<p>A <b>pyramid</b> is a polyhedron that has one base. The base can be any polygon. The other faces are triangles.</p> 

### EXAMPLE 1 Naming Prisms and Pyramids

Identify the bases and faces of each figure. Then name the figure.



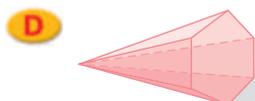
There are two rectangular bases.  
There are four other rectangular faces.  
The figure is a rectangular prism.



There is one rectangular base.  
There are four triangular faces.  
The figure is a rectangular pyramid.



There are two triangular bases.  
There are three rectangular faces.  
The figure is a triangular prism.

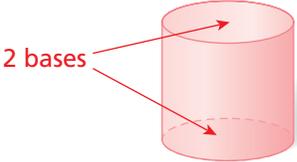
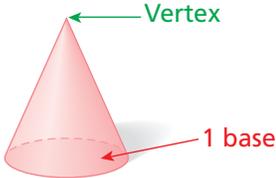


There is one hexagonal base.  
There are six triangular faces.  
The figure is a hexagonal pyramid.

### Remember!

A polygon with six sides is called a hexagon.

Other three-dimensional figures include *cylinders* and *cones*. These figures are not polyhedrons because they are not made of faces that are all polygons.

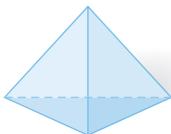
Cylinders	Cones
<p>A <b>cylinder</b> has two parallel, congruent bases that are circles.</p> 	<p>A <b>cone</b> has one base that is a circle and a surface that comes to a point called the vertex.</p> 

You can use properties to classify three-dimensional figures.

## EXAMPLE 2 Classifying Three-Dimensional Figures

Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

**A**



*The faces are all polygons, so the figure is a polyhedron.*

*There is one triangular base.*

The figure is a triangular pyramid.

**B**



*The faces are not all polygons, so the figure is not a polyhedron.*

*There are two circular bases.*

The figure is a cylinder.

**C**



*The faces are not all polygons, so the figure is not a polyhedron.*

*There is one circular base.*

The figure is a cone.

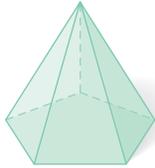
### Think and Discuss

- 1. Explain** how to identify a prism or a pyramid.
- 2. Compare and contrast** cylinders and prisms. How are they alike? How are they different?
- 3. Compare and contrast** pyramids and cones. How are they alike? How are they different?

**GUIDED PRACTICE**

See Example 1 Identify the bases and faces of each figure. Then name the figure.

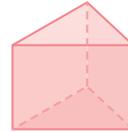
1.



2.



3.

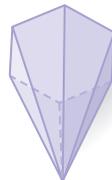


See Example 2 Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

4.



5.

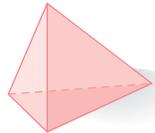


6.

**INDEPENDENT PRACTICE**

See Example 1 Identify the bases and faces of each figure. Then name the figure.

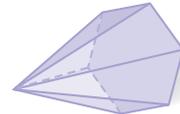
7.



8.



9.



See Example 2 Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

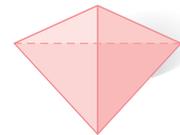
10.



11.



12.

**PRACTICE AND PROBLEM SOLVING****Extra Practice**

See page 746.

Identify the three-dimensional figure described.

13. two parallel, congruent square bases and four other polygonal faces
14. two parallel, congruent circular bases and one curved surface
15. one triangular base and three other triangular faces
16. one circular base and one curved surface

Name two examples of the three-dimensional figure described.

17. two parallel, congruent bases
18. one base

19. The structures in the photo at right are tombs of ancient Egyptian kings. No one knows exactly when the tombs were built, but some archaeologists think the first one might have been built around 2780 B.C.E. Name the shape of the ancient Egyptian structures.

2600 B.C.E.  
Ancient Egyptian structures at Giza



20. The Parthenon was built around 440 B.C.E. by the ancient Greeks. Its purpose was to house a statue of Athena, the Greek goddess of wisdom. Describe the three-dimensional shapes you see in the structure.



440 B.C.E.  
Parthenon

21. The Leaning Tower of Pisa began to lean as it was being built. To keep the tower from falling over, the upper sections (floors) were built slightly off center so that the tower would curve away from the way it was leaning. What shape is each section of the tower?



1173  
Leaning Tower of Pisa

22. **Challenge** The stainless steel structure at right, called the Unisphere, became the symbol of the New York World's Fair of 1964–1965. A sphere is a three-dimensional figure with a surface made up of all the points that are the same distance from a given point. Explain why the structure is not a sphere.



1964  
Unisphere

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Web Extra!

KEYWORD: MS7 Structures

## TEST PREP and Spiral Review

23. **Multiple Choice** Which figure has six rectangular faces?

(A) Rectangular prism

(C) Triangular pyramid

(B) Triangular prism

(D) Rectangular pyramid

24. **Multiple Choice** Which figure does NOT have two congruent bases?

(F) Cube

(G) Pyramid

(H) Prism

(J) Cylinder

Estimate each sum. (Lesson 3-7)

25.  $\frac{2}{5} + \frac{3}{8}$

26.  $\frac{1}{16} + \frac{4}{9}$

27.  $\frac{7}{9} + \frac{11}{12}$

28.  $\frac{1}{10} + \frac{1}{16}$

29. A store sells two sizes of detergent: 300 ounces for \$21.63 and 100 ounces for \$6.99. Which size detergent has the lowest price per ounce? (Lesson 5-2)



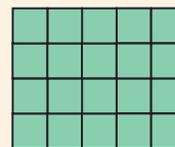
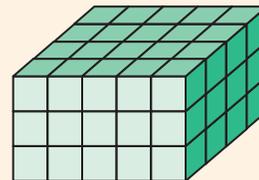
# Explore the Volume of Prisms and Cylinders



The volume of a three-dimensional figure is the number of cubes that it can hold. One cube represents one cubic unit of volume.

## Activity 1

- Use centimeter cubes to build the rectangular prism shown. What are the length, width, and height of the prism? How many cubes does the prism hold?
- You can find out how many cubes the prism holds without counting every cube. First look at the prism from above. How can you find the number of cubes in the top layer without counting every cube?
- Now look at the prism from the side. How many layers does the prism have? How can you use this to find the total number of cubes in the prism?



Top



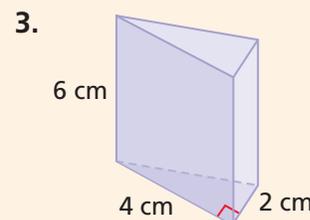
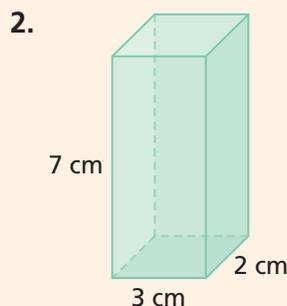
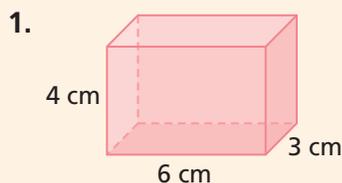
Side

## Think and Discuss

- Describe a shortcut for finding the number of cubes in a rectangular prism.
- Suppose you know the area of the base of a prism and the height of the prism. How can you find the prism's volume?
- Let the area of the base of a prism be  $B$  and the height of the prism be  $h$ . Write a formula for the prism's volume  $V$ .

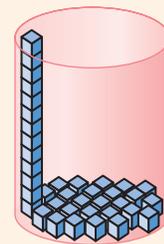
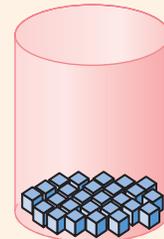
## Try This

Use the formula you discovered to find the volume of each prism.



## Activity 2

- 1 You can use a process similar to that in Activity 1 to develop the formula for the volume of a cylinder. You will need an empty soup can or other cylindrical container. Remove one of the bases.
- 2 Arrange centimeter cubes in a single layer at the bottom of the cylinder. Fit as many cubes into the layer as possible. How many cubes are in this layer?
- 3 To find how many layers of cubes would fit in the cylinder, make a stack of cubes along the inside of the cylinder. How many layers would fit in the cylinder?
- 4 How can you use what you know to find the approximate number of cubes that would fit in the cylinder?

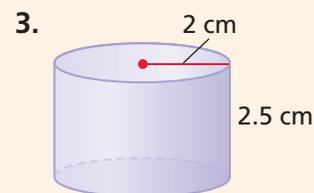
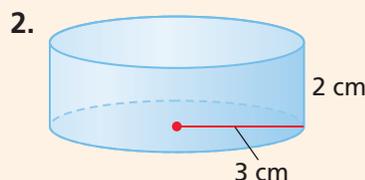
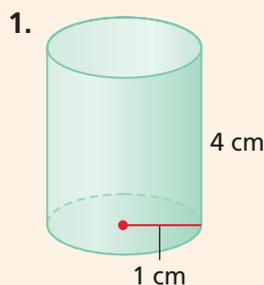


## Think and Discuss

1. Suppose you know the area of the base of a cylinder and the height of the cylinder. How can you find the cylinder's volume?
2. Let the area of the base of a cylinder be  $B$  and the height of the cylinder be  $h$ . Write a formula for the cylinder's volume  $V$ .
3. The base of a cylinder is a circle with radius  $r$ . How can you find the area of the base? How can you use this in your formula for the volume of a cylinder?

## Try This

Use the formula you discovered to find the volume of each cylinder. Use 3.14 for  $\pi$  and round to the nearest tenth.



# 10-2

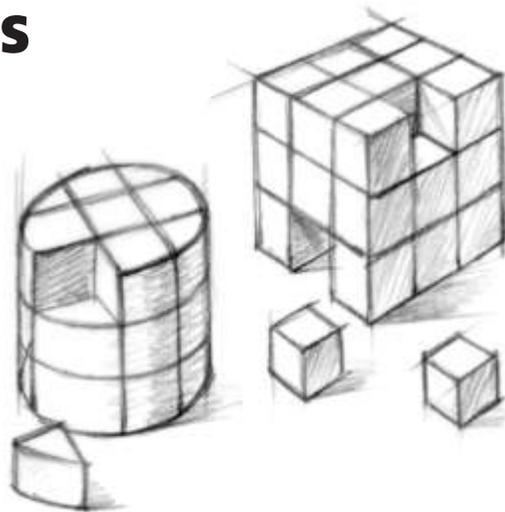
## Volume of Prisms and Cylinders

**Learn** to find the volume of prisms and cylinders.

### Vocabulary

**volume**

Any three-dimensional figure can be filled completely with congruent cubes and parts of cubes. The **volume** of a three-dimensional figure is the number of cubes it can hold. Each cube represents a unit of measure called a cubic unit.



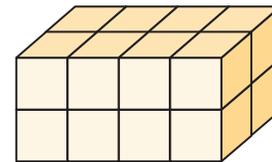
### EXAMPLE 1 Using Cubes to Find the Volume of a Rectangular Prism

Find how many cubes the prism holds. Then give the prism's volume.

You can find the volume of this prism by counting how many cubes tall, long, and wide the prism is and then multiplying.

$$2 \cdot 4 \cdot 2 = 16$$

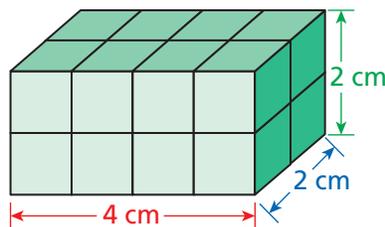
There are 16 cubes in the prism, so the volume is 16 cubic units.



### Reading Math

Any unit of measurement with an exponent of 3 is a cubic unit. For example,  $m^3$  means "cubic meter," and  $in^3$  means "cubic inch."

To find a prism's volume, multiply its length by its width by its height.



$$4 \text{ cm} \cdot 2 \text{ cm} \cdot 2 \text{ cm} = 16 \text{ cm}^3$$

$\text{length} \cdot \text{width} \cdot \text{height} = \text{volume}$   
 $\text{area of base} \cdot \text{height} = \text{volume}$

The volume of a rectangular prism is the area of its base times its height. This formula can be used to find the volume of any prism.

### VOLUME OF A PRISM

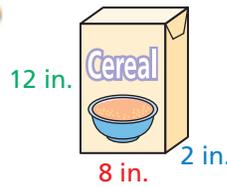
The volume  $V$  of a prism is the area of its base  $B$  times its height  $h$ .

$$V = Bh$$

## EXAMPLE 2 Using a Formula to Find the Volume of a Prism

Find the volume of each figure.

**A**



$$V = Bh$$

Use the formula.

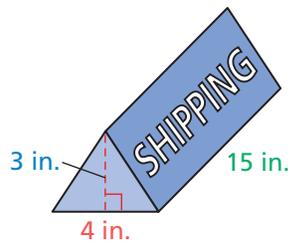
The base is a rectangle:  $B = 8 \cdot 2 = 16$ .

$$V = 16 \cdot 12$$
 Substitute for  $B$  and  $h$ .

$$V = 192$$
 Multiply.

The volume of the cereal box is  $192 \text{ in}^3$ .

**B**



$$V = Bh$$

Use the formula.

The base is a triangle:

$$B = \frac{1}{2} \cdot 4 \cdot 3 = 6.$$

$$V = 6 \cdot 15$$
 Substitute for  $B$  and  $h$ .

$$V = 90$$

Multiply.

The volume of the shipping carton is  $90 \text{ in}^3$ .

Finding the volume of a cylinder is similar to finding the volume of a prism.

### VOLUME OF A CYLINDER

The volume  $V$  of a cylinder is the area of its base  $B$  times its height  $h$ .

$$V = Bh \quad \text{or} \quad V = \pi r^2 h, \text{ where } B = \pi r^2$$

## EXAMPLE 3 Using a Formula to Find the Volume of a Cylinder

A can of shoe polish is shaped like a cylinder. Find its volume to the nearest tenth. Use 3.14 for  $\pi$ .

$$V = Bh$$

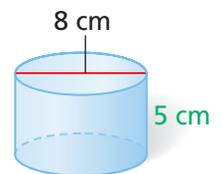
Use the formula.

The base is a circle:  $B = \pi \cdot 4^2 \approx 50.24 \text{ cm}^2$ .

$$V \approx 50.24 \cdot 5$$
 Substitute for  $B$  and  $h$ .

$$V \approx 251.2$$
 Multiply.

The volume of the shoe polish can is about  $251.2 \text{ cm}^3$ .

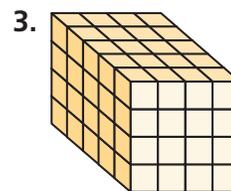
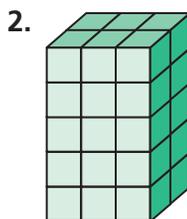
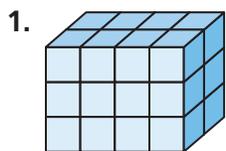


### Think and Discuss

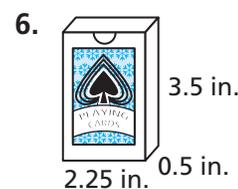
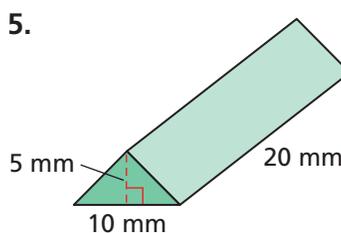
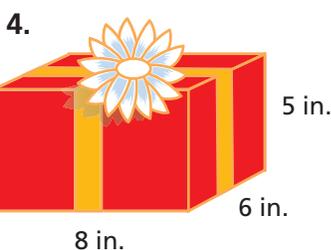
- 1. Explain** what a cubic unit is. What units would you use for the volume of a figure measured in yards?
- 2. Compare and contrast** the formulas for volume of a prism and volume of a cylinder. How are they alike? How are they different?

## GUIDED PRACTICE

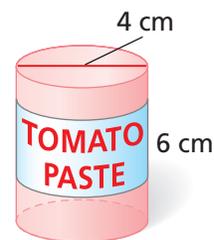
See Example 1 Find how many cubes each prism holds. Then give the prism's volume.



See Example 2 Find the volume of each figure.

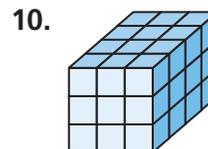
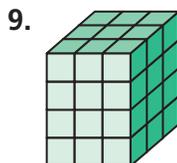
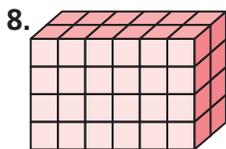


See Example 3 7. A can of tomato paste is shaped like a cylinder. It is 4 cm wide and 6 cm tall. Find its volume to the nearest tenth. Use 3.14 for  $\pi$ .

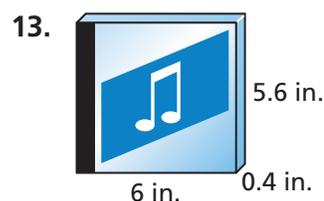
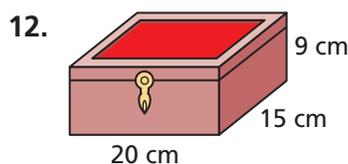
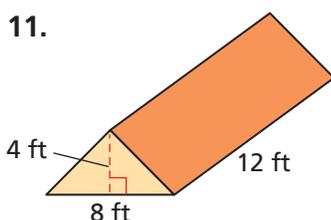


## INDEPENDENT PRACTICE

See Example 1 Find how many cubes each prism holds. Then give the prism's volume.



See Example 2 Find the volume of each figure.



See Example 3 14. A paper towel roll is shaped like a cylinder. It is 4 cm wide and 28 cm tall. Find its volume to the nearest tenth. Use 3.14 for  $\pi$ .

## PRACTICE AND PROBLEM SOLVING

### Extra Practice

See page 746.



### Life Science



Marine biologists insert tags containing tiny microchips into the bellies of salmon to study the migration patterns of these fish.

go.hrw.com

Web Extra!

KEYWORD: MS7 Tags

15. **Multi-Step** The base of a triangular prism is a right triangle with hypotenuse 10 m long and one leg 6 m long. If the height of the prism is 12 m, what is the volume of the prism?

16. **Life Science** An ID tag containing a microchip can be injected into a pet, such as a dog or cat. These microchips are cylindrical and can be as small as 12 mm in length and 2.1 mm in diameter. Use rounding to estimate the volume of one of these microchips. Then find the volume to the nearest tenth. Use 3.14 for  $\pi$ .

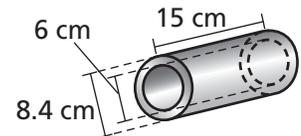
17. **Recreation** The tent shown is in the shape of a triangular prism. How many cubic feet of space are in the tent?



18. **What's the Error?** A student said the volume of a cylinder with a 3-inch diameter is two times the volume of a cylinder with the same height and a 1.5-inch radius. What is the error?

19. **Write About It** Explain the similarities and differences between finding the volume of a cylinder and finding the volume of a triangular prism.

20. **Challenge** Find the volume, to the nearest tenth, of the material that makes up the pipe shown. Use 3.14 for  $\pi$ .

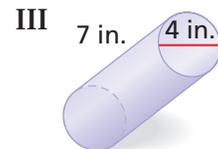
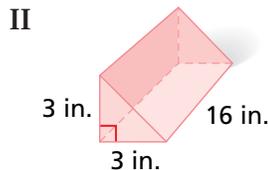
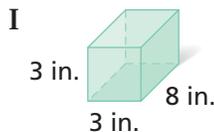


## TEST PREP and Spiral Review

21. **Multiple Choice** What is the volume of a triangular prism that is 10 in. long, 7 in. wide, and 4 in. high?

(A)  $110 \text{ in}^2$       (B)  $140 \text{ in}^2$       (C)  $205 \text{ in}^2$       (D)  $280 \text{ in}^2$

22. **Multiple Choice** Which figures have the same volume?



(F) I and II      (G) I and III      (H) II and III      (J) I, II, and III

Find the simple interest. (Lesson 6-7)

23.  $P = \$3,600$ ;  $r = 5\%$ ;  $t = 1.5$  years

24.  $P = \$10,000$ ;  $r = 3.2\%$ ;  $t = 2$  years

25. Students collected data on the number of visitors to an amusement park over a period of 30 days. Choose the type of graph that would best represent the data. (Lesson 7-7)

# 10-3 Volume of Pyramids and Cones

**Learn** to find the volume of pyramids and cones.

Suppose you have a square-pyramid-shaped container and a square-prism-shaped container, and the bases and heights are the same size. If you pour sand from the pyramid into the prism, it appears that the prism holds three times as much sand as the pyramid.

In fact, the volume of a pyramid is exactly one-third the volume of a prism if they have the same height and same-size base.

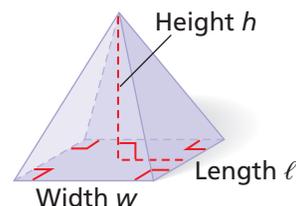
The height of a pyramid is the perpendicular distance from the pyramid's base to its vertex.



## VOLUME OF A RECTANGULAR PYRAMID

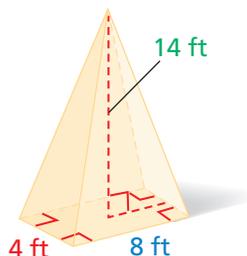
The volume  $V$  of a rectangular pyramid is one-third the area of its base  $B$  times its height  $h$ .

$$V = \frac{1}{3}Bh \quad \text{or} \quad V = \frac{1}{3}\ell wh, \text{ where } B = \ell w$$



### EXAMPLE 1 Finding the Volume of a Rectangular Pyramid

Find the volume of the pyramid to the nearest tenth. Estimate to check whether the answer is reasonable.



$$V = \frac{1}{3}Bh$$

Use the formula.

The base is a rectangle, so

$$B = 4 \cdot 8 = 32.$$

Substitute for  $B$  and  $h$ .

$$V = \frac{1}{3} \cdot 32 \cdot 14$$

$$V \approx 149.3 \text{ ft}^3$$

Multiply.

$$\begin{aligned} \text{Estimate } V &\approx \frac{1}{3} \cdot 30 \cdot 15 \\ &= 150 \text{ ft}^3 \end{aligned}$$

Round the measurements.

The answer is reasonable.

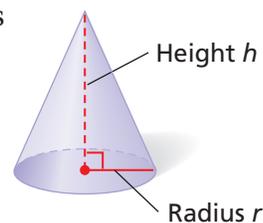
Similar to the relationship between volumes of prisms and pyramids, the volume of a cone is one-third the volume of a cylinder with the same height and a congruent base.

The height of a cone is the perpendicular distance from the cone's base to its vertex.

### VOLUME OF A CONE

The volume  $V$  of a cone is one-third the area of its base  $B$  times its height  $h$ .

$$V = \frac{1}{3}Bh \quad \text{or} \quad V = \frac{1}{3}\pi r^2h, \text{ where } B = \pi r^2$$

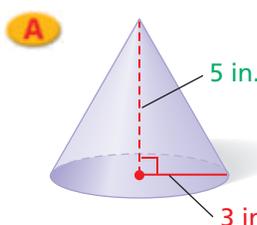


### EXAMPLE 2 Finding the Volume of a Cone

#### Helpful Hint

To estimate the volume of a cone, round  $\pi$  to 3 so that  $\frac{1}{3} \cdot \pi$  becomes  $\frac{1}{3} \cdot 3$ , which is 1.

Find the volume of each cone to the nearest tenth. Use 3.14 for  $\pi$ . Estimate to check whether the answer is reasonable.



$$V = \frac{1}{3}Bh$$

Use the formula.

The base is a circle, so  $B = \pi \cdot r^2 = 3.14 \cdot 3^2 \approx 28.27$ .

$$V \approx \frac{1}{3} \cdot 28.27 \cdot 5$$

Substitute for  $B$  and  $h$ .

$$V \approx 47.1 \text{ in}^3$$

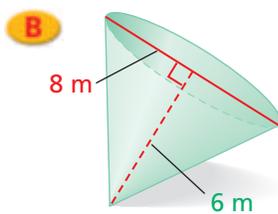
Multiply.

Estimate  $V \approx \left(\frac{1}{3} \cdot \pi\right) 3^2 \cdot 5$

$$\frac{1}{3} \cdot \pi \approx 1$$

$$\approx 45 \text{ in}^3$$

The answer is reasonable.



$$V = \frac{1}{3}Bh$$

Use the formula.

The base is a circle, so  $B = \pi \cdot r^2 = 3.14 \cdot \left(\frac{8}{2}\right)^2 \approx 50.3$ .

$$V \approx \frac{1}{3} \cdot 50.3 \cdot 6$$

Substitute for  $B$  and  $h$ .

$$V \approx 100.6 \text{ m}^2$$

Multiply.

Estimate  $V \approx \left(\frac{1}{3} \cdot \pi\right) 4^2 \cdot 6$

$$\frac{1}{3} \cdot \pi \approx 1$$

$$\approx 96 \text{ m}^2$$

The answer is reasonable.

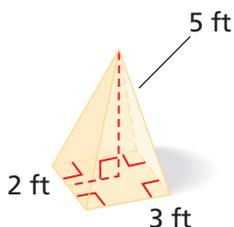
### Think and Discuss

- 1. Explain** how to find the volume of a cone given the diameter of the base and the height of the cone.
- 2. Compare and contrast** the formulas for volume of a pyramid and volume of a cone. How are they alike? How are they different?

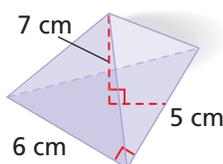
## GUIDED PRACTICE

See Example 1 Find the volume of each pyramid to the nearest tenth. Estimate to check whether the answer is reasonable.

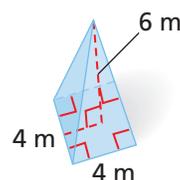
1.



2.

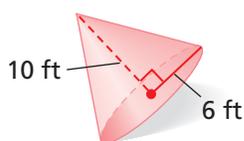


3.

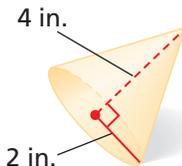


See Example 2 Find the volume of each cone to the nearest tenth. Use 3.14 for  $\pi$ . Estimate to check whether the answer is reasonable.

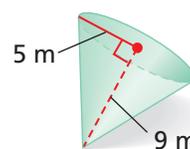
4.



5.



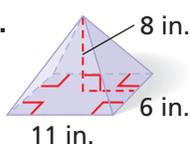
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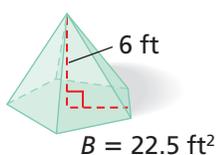
## INDEPENDENT PRACTICE

See Example 1 Find the volume of each pyramid to the nearest tenth. Estimate to check whether the answer is reasonable.

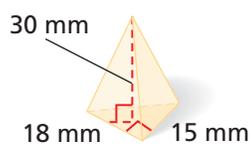
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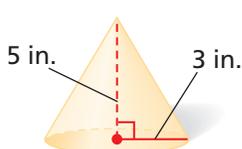


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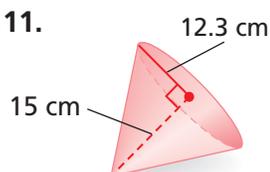


See Example 2 Find the volume of each cone to the nearest tenth. Use 3.14 for  $\pi$ . Estimate to check whether the answer is reasonable.

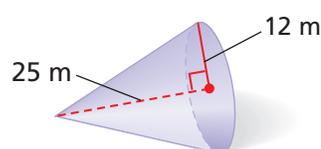
10.



11.



12.



## PRACTICE AND PROBLEM SOLVING

## Extra Practice

See page 746.

Find the volume of each figure to the nearest tenth. Use 3.14 for  $\pi$ .

13. a 7 ft tall rectangular pyramid with base 4 ft by 5 ft

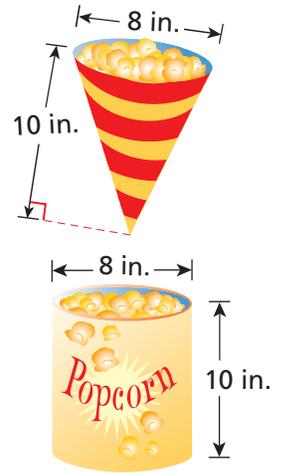
14. a cone with radius 8 yd and height 12 yd

15. **Multi-Step** Find the volume of an 8 in. tall right triangular pyramid with a base hypotenuse of 5 in. and base leg of 3 in.

16. **Architecture** The steeple on a building is a square pyramid with a base area of 12 square feet and a height of 15 feet. How many cubic feet of concrete was used to make the steeple?

17. **Multi-Step** A snack bar sells popcorn in the containers shown at right.

- Based on the formulas for volume of a cylinder and a cone, how many times as much popcorn does the larger container hold?
- How many cubic inches of popcorn, to the nearest tenth, does the cone-shaped container hold? Use 3.14 for  $\pi$ .
- How many cubic inches of popcorn does the cylinder-shaped container hold? Use 3.14 for  $\pi$ .
- Do your answers to parts **b** and **c** confirm your answer to part **a**? If not, find the error.



18. **Critical Thinking** Write a proportion of volumes for the given figures.

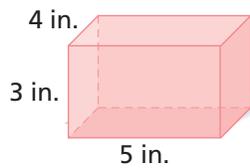


Figure 1

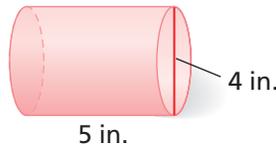


Figure 2

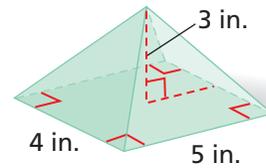


Figure 3

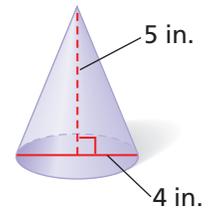


Figure 4

- What's the Question?** The answer is: The volume of figure A is  $\frac{1}{3}$  the volume of figure B. What's the question?
- Write About It** Compare finding the volume of a cylinder with finding the volume of a cone that has the same height and base.
- Challenge** What effect does doubling the radius of a cone's base have on the cone's volume?



### TEST PREP and Spiral Review

- Multiple Choice** Which is the best estimate for the volume of a cone with a radius of 5 cm and a height of 8 cm?  
 (A)  $40 \text{ cm}^3$       (B)  $80 \text{ cm}^3$       (C)  $200 \text{ cm}^3$       (D)  $800 \text{ cm}^3$
- Short Response** A rectangular prism and a square pyramid both have a square base with side lengths of 5 inches and heights of 7 inches. Find the volume of each figure. Then explain the relationship between the volume of the prism and the volume of the pyramid.

Name the types of quadrilaterals that have each property. (Lesson 8-7)

- four congruent sides
- two sets of parallel sides

Find the volume of each figure to the nearest tenth. Use 3.14 for  $\pi$ . (Lesson 10-2)

- cylinder:  $d = 6 \text{ m}$ ,  $h = 8 \text{ m}$
- triangular prism:  $B = 22 \text{ ft}^2$ ,  $h = 5 \text{ ft}$

### Quiz for Lessons 10-1 Through 10-3

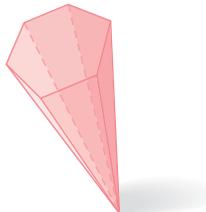
#### 10-1 Introduction to Three-Dimensional Figures

Classify each figure as a polyhedron or not a polyhedron. Then name the figure.

1.



2.



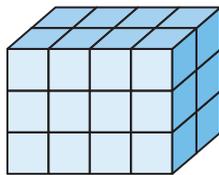
3.



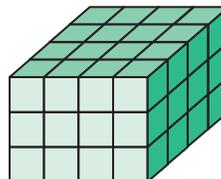
#### 10-2 Volume of Prisms and Cylinders

Find how many cubes each prism holds. Then give the prism's volume.

4.



5.



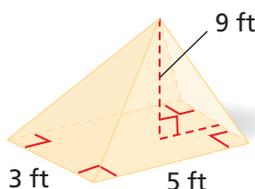
6. A box is shaped like a rectangular prism. It is 6 ft long, 2 ft wide, and 3 ft high. Find its volume.

7. A can is shaped like a cylinder. It is 5.2 cm wide and 2.3 cm tall. Find its volume. Use 3.14 for  $\pi$ .

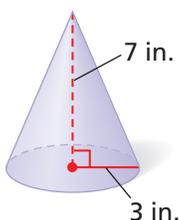
#### 10-3 Volume of Pyramids and Cones

Find the volume of each figure to the nearest tenth. Use 3.14 for  $\pi$ .

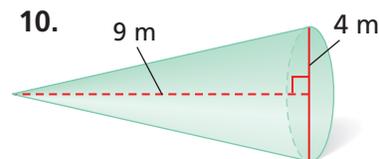
8.



9.



10.



11. A cone has a radius of 2.5 cm and a height of 14 cm. What is the volume of the cone to the nearest hundredth? Use 3.14 for  $\pi$ .

# Focus on Problem Solving



## Solve

- Choose an operation

When choosing an operation to use when solving a problem, you need to decide which action the problem is asking you to take. If you are asked to combine numbers, then you need to add. If you are asked to take away numbers or to find the difference between two numbers, then you need to subtract. You need to use multiplication when you put equal parts together and division when you separate something into equal parts.

## Determine the action in each problem. Then tell which operation should be used to solve the problem. Explain your choice.

- 1 Jeremy filled a sugar cone completely full of frozen yogurt and then put one scoop of frozen yogurt on top. The volume of Jeremy's cone is about  $20.93 \text{ in}^3$ , and the volume of the scoop that Jeremy used is about  $16.75 \text{ in}^3$ . About how much frozen yogurt, in cubic inches, did Jeremy use?
- 2 The volume of a cylinder equals the combined volumes of three cones that each have the same height and base size as the cylinder. What is the volume of a cylinder if a cone of the same height and base size has a volume of  $45.2 \text{ cm}^3$ ?
- 3 The biology class at Jefferson High School takes care of a family of turtles that is kept in a glass tank with water, rocks, and plants. The volume of the tank is 2.75 cubic feet. At the end of the year, the baby turtles will have grown and will be moved into a tank that is 6.15 cubic feet. How much greater will the volume of the new tank be than that of the old tank?
- 4 Brianna is adding a second section to her hamster cage. The two sections will be connected by a tunnel that is made of 4 cylindrical parts, all the same size. If the volume of the tunnel is 56.52 cubic inches, what is the volume of each part of the tunnel?





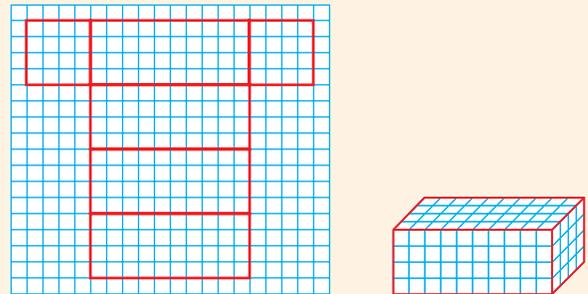
# Use Nets to Build Prisms and Cylinders



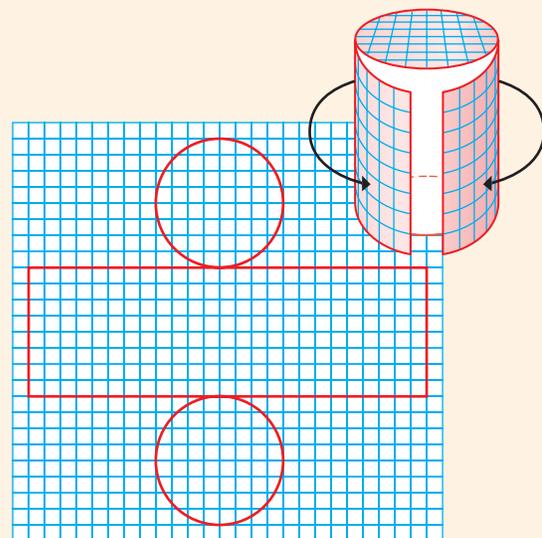
A net is a pattern of two-dimensional figures that can be folded to make a three-dimensional figure. You can use  $\frac{1}{4}$ -inch graph paper to help you make nets.

## Activity

- 1 Use a net to construct a rectangular prism.
  - a. Draw the net at right on a piece of graph paper. Each rectangle is 10 squares by 4 squares. The two squares are 4 small squares on each side.
  - b. Cut out the net. Fold the net along the edges of each rectangle to make a rectangular prism. Tape the edges to hold them in place.



- 2 Use a net to construct a cylinder.
  - a. Draw the net at right on a piece of graph paper. The rectangle is 25 squares by 8 squares. Use a compass to make the circles. Each circle has a radius of 4 squares.
  - b. Cut out the net. Fold the net as shown to make a cylinder. Tape the edges to hold them in place.



## Think and Discuss

1. What are the dimensions, in inches, of the rectangular prism that you built?
2. What is the height, in inches, of the cylinder that you built? What is the cylinder's radius?

## Try This

1. Use a net to construct a rectangular prism that is 1 inch by 2 inches by 3 inches.
2. Use a net to construct a cylinder with a height of 1 inch and a radius of  $\frac{1}{2}$  in. (*Hint:* The length of the rectangle in the net must match the circumference of the circles, so the length should be  $2\pi r = 2\pi(\frac{1}{2}) \approx 3.14$  inches.)

# 10-4

## Surface Area of Prisms and Cylinders

**Learn** to find the surface area of prisms and cylinders.

### Vocabulary

**net**

**surface area**

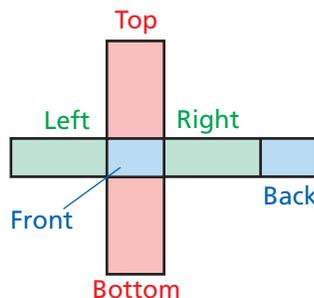
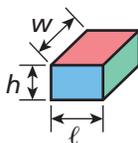
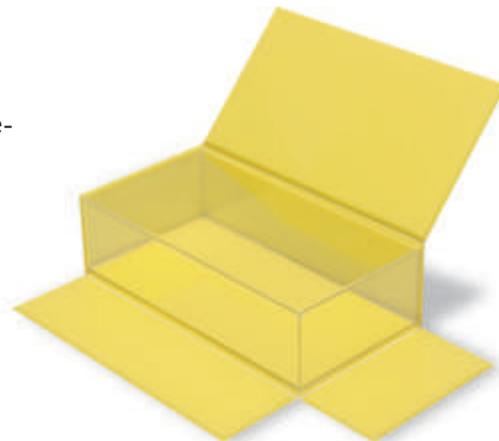
If you remove the surface from a three-dimensional figure and lay it out flat, the pattern you make is called a **net**.

Nets allow you to see all the surfaces of a solid at one time. You can use nets to help you find the *surface area* of a three-dimensional figure.

**Surface area** is the sum of the areas of all of the surfaces of a figure.

You can use nets to write formulas for the surface area of prisms. The surface area  $S$  of a prism is the sum of the areas of the faces of the prism. For the rectangular prism shown:

$$S = \ell w + \ell h + wh + \ell w + \ell h + wh = 2\ell w + 2\ell h + 2wh$$



### SURFACE AREA OF A RECTANGULAR PRISM

The surface area of a rectangular prism is the sum of the areas of each face.

$$S = 2\ell w + 2\ell h + 2wh$$

### EXAMPLE 1 Finding the Surface Area of a Prism

Find the surface area of the prism formed by the net.

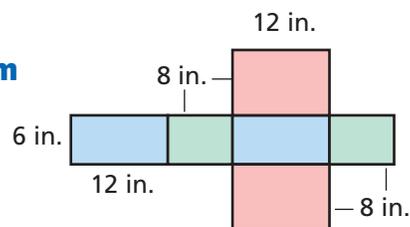
$$S = 2\ell w + 2\ell h + 2wh$$

$$S = (2 \cdot 12 \cdot 8) + (2 \cdot 12 \cdot 6) + (2 \cdot 8 \cdot 6)$$

$$S = 192 + 144 + 96$$

$$S = 432$$

The surface area of the prism is  $432 \text{ in}^2$ .



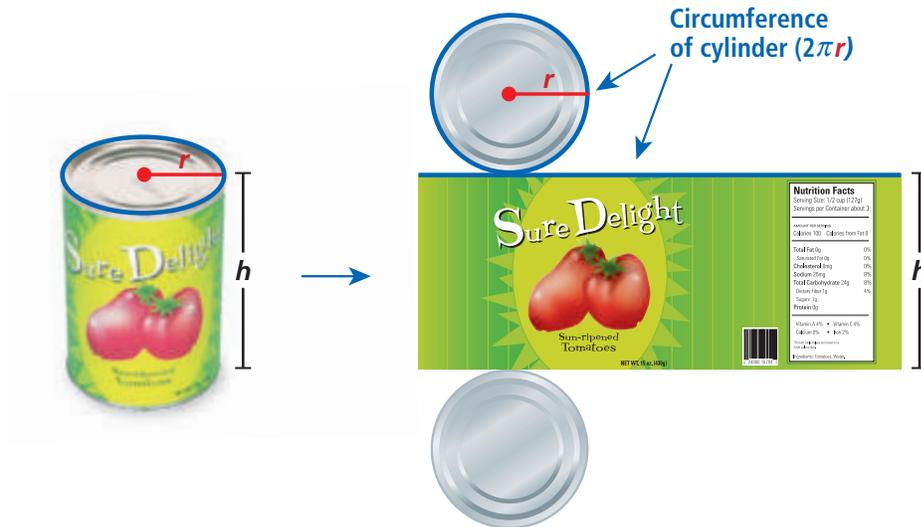
*Substitute.*

*Multiply.*

*Add.*

If you could remove the curved surface from a cylinder, like peeling a label from a can, you would see that it has the shape of a rectangle when flattened out.

You can draw a net for a cylinder by drawing the circular bases (like the ends of a can) and the rectangular curved surface as shown below. The length of the rectangle is the circumference,  $2\pi r$ , of the base of the cylinder. So the area of the curved surface is  $2\pi r \cdot h$ . The area of each base is  $\pi r^2$ .



$$\begin{aligned} \text{Surface area} &= \text{area of top} + \text{area of bottom} + \text{area of curved surface} \\ &= \pi r^2 + \pi r^2 + (2\pi r)h \\ &= 2\pi r^2 + 2\pi rh \end{aligned}$$

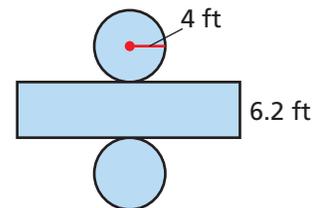
### SURFACE AREA OF A CYLINDER

The surface area  $S$  of a cylinder is the sum of the areas of its bases,  $2\pi r^2$ , plus the area of its curved surface,  $2\pi rh$ .

$$S = 2\pi r^2 + 2\pi rh$$

#### EXAMPLE 2 Finding the Surface Area of a Cylinder

Find the surface area of the cylinder formed by the net to the nearest tenth. Use 3.14 for  $\pi$ .



$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ S &\approx (2 \cdot 3.14 \cdot 4^2) + (2 \cdot 3.14 \cdot 4 \cdot 6.2) \\ S &\approx 100.48 + 155.744 \\ S &\approx 256.224 \\ S &\approx 256.2 \end{aligned}$$

Use the formula.

Substitute.

Multiply.

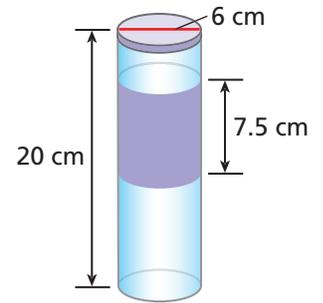
Add.

Round.

The surface area of the cylinder is about  $256.2 \text{ ft}^2$ .

**EXAMPLE 3** PROBLEM SOLVING APPLICATION

What percent of the total surface area of the tennis ball can is covered by the label? Use 3.14 for  $\pi$ .

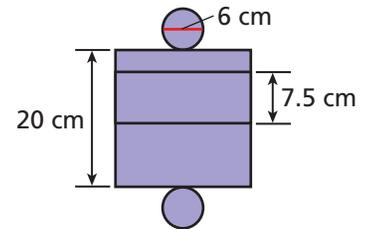
**1 Understand the Problem**

List the **important information**:

- The can is approximately cylinder-shaped.
- The height of the can is 20 cm.
- The diameter of the can is 6 cm.
- The height of the label is 7.5 cm.

**2 Make a Plan**

Find the surface area of the can and the area of the label. Divide to find the percent of the surface area covered by the label.

**3 Solve**

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &\approx 2(3.14)(3)^2 + 2(3.14)(3)(20) && \text{Substitute for } r \text{ and } h. \\ &\approx 433.32 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= \ell w \\ &= (2\pi r)w && \text{Substitute } 2\pi r \text{ for } \ell. \\ &\approx 2(3.14)(3)(7.5) && \text{Substitute for } r \text{ and } w. \\ &\approx 141.3 \text{ cm}^2 \end{aligned}$$

Percent of the surface area covered by the label:  $\frac{141.3 \text{ cm}^2}{433.32 \text{ cm}^2} \approx 32.6\%$ .  
About 32.6% of the can's surface area is covered by the label.

**4 Look Back**

Estimate and compare the areas of the two rectangles in the net.

$$\begin{aligned} \text{Label: } 2(3)(3)(8) &= 144 \text{ cm}^2 && \frac{144 \text{ cm}^2}{360 \text{ cm}^2} = 40\% \\ \text{Can: } 2(3)(3)(20) &= 360 \text{ cm}^2 \end{aligned}$$

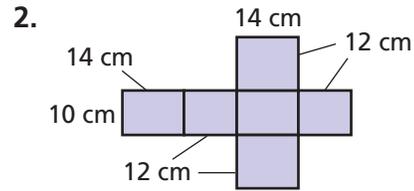
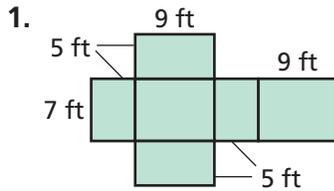
The answer should be less than 40% because you did not consider the area of the two circles. So 32.6% is reasonable.

**Think and Discuss**

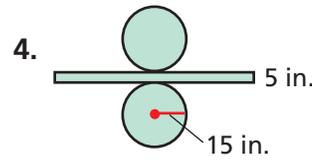
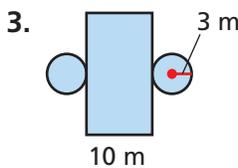
- 1. Explain** how you would find the surface area of an open-top box that is shaped like a rectangular prism.
- 2. Describe** the shapes in a net used to cover a cylinder.

**GUIDED PRACTICE**

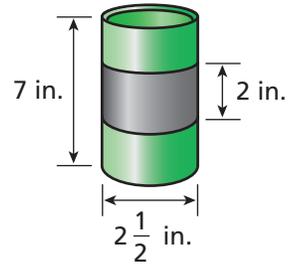
See Example 1 Find the surface area of the prism formed by each net.



See Example 2 Find the surface area of the cylinder formed by each net to the nearest tenth. Use 3.14 for  $\pi$ .

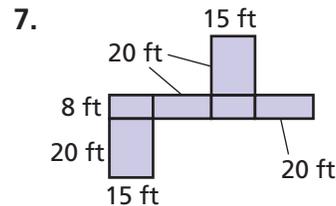
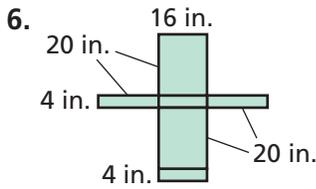


See Example 3 5. A travel mug is cylindrical, and its  $2\frac{1}{2}$  in. width fits into most drink holders. What percent of the total surface area of the mug is covered by the 2 in. wide grip? Use 3.14 for  $\pi$ .

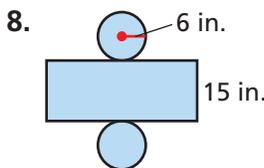


**INDEPENDENT PRACTICE**

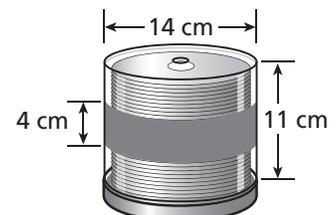
See Example 1 Find the surface area of the prism formed by each net.



See Example 2 Find the surface area of the cylinder formed by each net to the nearest tenth. Use 3.14 for  $\pi$ .



See Example 3 10. A stack of DVDs sits on a base and is covered by an 11 cm tall cylindrical lid. What percent of the surface area of the lid is covered by the label? Use 3.14 for  $\pi$ . (Hint: The lid has no bottom.)







# Investigate the Surface Areas of Similar Prisms

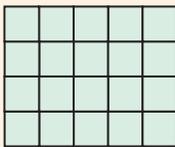
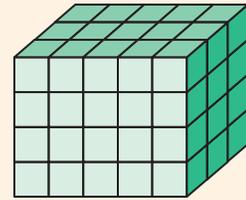
Use with Lesson 10-5



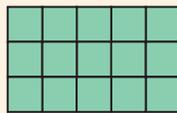
Recall that the surface area of a three-dimensional figure is the sum of the areas of all of its surfaces. You can use centimeter cubes to explore the surface areas of prisms.

## Activity 1

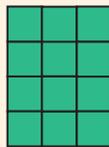
- Use centimeter cubes to build the rectangular prism shown here.
- You can find the surface area of the prism by first finding the areas of its front face, top face, and side face. Look at the prism from each of these views. Count the exposed cube faces to find the area of each face of the prism. Record the areas in the table.



Front



Top



Side

	Front Face	Top Face	Side Face
Area	■	■	■

- Find the surface area of the prism as follows:  

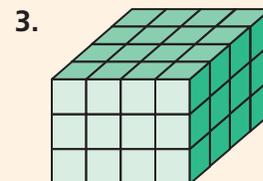
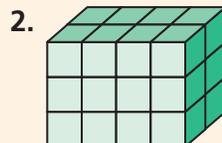
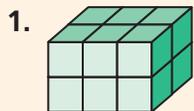
$$\text{surface area} = 2 \cdot (\text{area of front face}) + 2 \cdot (\text{area of top face}) + 2 \cdot (\text{area of side face})$$

## Think and Discuss

- Why do you multiply the areas of the front face, top face, and side face by 2 to find the surface area of the prism?
- What are the length, width, and height of the prism in centimeters? What surface area do you get when you use the formula  $S = 2\ell w + 2\ell h + 2wh$ ?

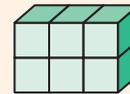
## Try This

Use centimeter cubes to build each prism. Then find its surface area.



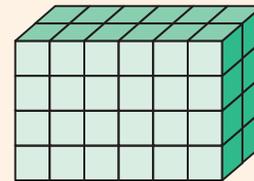
## Activity 2

- 1 Use centimeter cubes to build rectangular prism  $A$  as shown.



Prism  $A$

- 2 Now use centimeter cubes to build a prism  $B$  that is similar to prism  $A$  by a scale factor of 2. Each dimension of the new prism should be 2 times the corresponding dimension of prism  $A$ .



Prism  $B$

- 3 Use the method from Activity 1 to find the areas of the front face, top face, and side face of each prism. Record the areas in the table.

	Area of Front Face	Area of Top Face	Area of Side Face
Prism $A$	■	■	■
Prism $B$	■	■	■

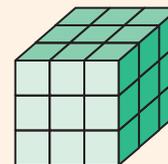
- 4 Find the surface area of prism  $A$  and the surface area of prism  $B$ .
- 5 Repeat the above process, this time building a prism  $C$  that is larger than prism  $A$  by a scale factor of 3. Add a row to your table for prism  $C$ , and find the areas of the front face, top face, and side face of prism  $C$ .

## Think and Discuss

- In 4, how does the surface area of prism  $B$  compare with the surface area of prism  $A$ ? How is this related to the scale factor?
- In 5, how does the surface area of prism  $C$  compare with the surface area of prism  $A$ ? How is this related to the scale factor?
- Suppose three-dimensional figure  $Y$  is similar to three-dimensional figure  $X$  by a scale factor of  $k$ . How are the surface areas related?

## Try This

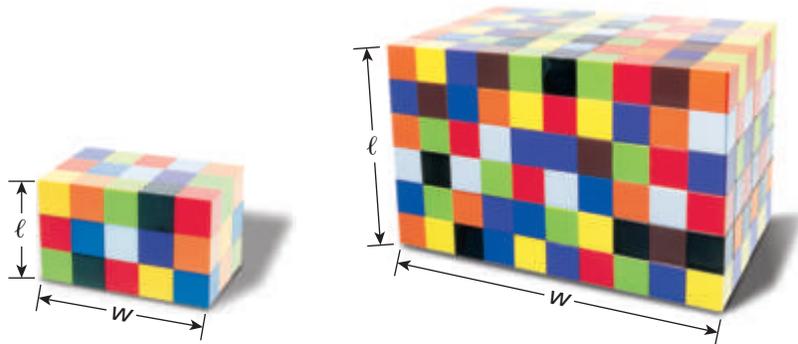
- Find the surface area of prism  $R$ .
- Prism  $S$  is larger than prism  $R$  by a scale factor of 4. Use what you discovered to find the surface area of prism  $S$ .



Prism  $R$

# 10-5 Changing Dimensions

**Learn** to find the volume and surface area of similar three-dimensional figures. Recall that similar figures have proportional side lengths. The surface areas of similar three-dimensional figures are also proportional. To see this relationship, you can compare the areas of corresponding faces of similar rectangular prisms.



### Remember!

A scale factor is a number that every dimension of a figure is multiplied by to make a similar figure.

Area of front of smaller prism

$$\begin{aligned} \ell \cdot w \\ 3 \cdot 5 \\ \mathbf{15} \end{aligned}$$

Area of front of larger prism

$$\begin{aligned} \ell \cdot w \\ 6 \cdot 10 \\ (3 \cdot 2) \cdot (5 \cdot 2) \quad \leftarrow \text{Each dimension is multiplied by a scale factor of } 2. \\ (3 \cdot 5) \cdot (2 \cdot 2) \\ \mathbf{15 \cdot 2^2} \end{aligned}$$

The area of the front face of the larger prism is  $2^2$  times the area of the front face of the smaller prism. This is true for the entire surface area of the prisms.

### SURFACE AREA OF SIMILAR FIGURES

If three-dimensional figure  $B$  is similar to figure  $A$  by a scale factor, then the surface area of  $B$  is equal to the surface area of  $A$  times the square of the scale factor.

$$\text{surface area of figure } A = \text{surface area of figure } B \cdot (\text{scale factor})^2$$

### EXAMPLE 1 Finding the Surface Area of a Similar Figure

**A** The surface area of a box is  $27 \text{ in}^2$ . What is the surface area of a similar box that is larger by a scale factor of 5?

$$S = 27 \cdot 5^2 \quad \text{Multiply by the square of the scale factor.}$$

$$S = 27 \cdot 25 \quad \text{Evaluate the power.}$$

$$S = 675 \text{ in}^2 \quad \text{Multiply.}$$

**B** The surface area of the Great Pyramid was originally  $1,160,280 \text{ ft}^2$ . What is the surface area, to the nearest tenth, of a model of the pyramid that is smaller by a scale factor of  $\frac{1}{500}$ ?

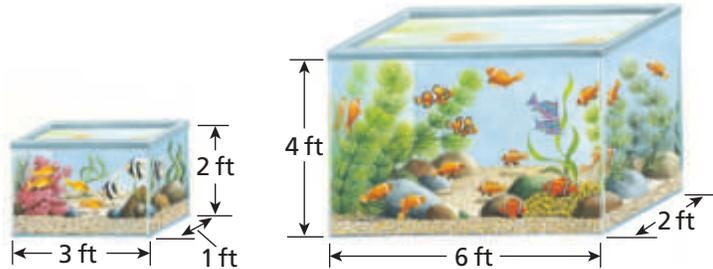
$$S = 1,160,280 \cdot \left(\frac{1}{500}\right)^2 \quad \text{Multiply by the square of the scale factor.}$$

$$S = 1,160,280 \cdot \frac{1}{250,000} \quad \text{Evaluate the power.}$$

$$S = 4.64112 \quad \text{Multiply.}$$

$$S \approx 4.6 \text{ ft}^2$$

The volumes of similar three-dimensional figures are also related.



Volume of smaller tank

$$\begin{aligned} \ell \cdot w \cdot h \\ 2 \cdot 3 \cdot 1 \\ 6 \end{aligned}$$

Volume of larger tank

$$\begin{aligned} \ell \cdot w \cdot h \\ 4 \cdot 6 \cdot 2 \\ (2 \cdot 2) \cdot (3 \cdot 2) \cdot (1 \cdot 2) \\ (2 \cdot 3 \cdot 1) \cdot (2 \cdot 2 \cdot 2) \\ 6 \cdot 2^3 \end{aligned}$$

Each dimension has a scale factor of 2.

**Remember!**

$$2 \cdot 2 \cdot 2 = 2^3$$

The volume of the larger tank is  $2^3$  times the volume of the smaller tank.

### VOLUME OF SIMILAR FIGURES

If three-dimensional figure  $B$  is similar to figure  $A$  by a scale factor, then the volume of  $B$  is equal to the volume of  $A$  times the cube of the scale factor.

$$\text{volume of figure } A = \text{volume of figure } B \cdot (\text{scale factor})^3$$

### EXAMPLE 2 Finding Volume Using Similar Figures

The volume of a bucket is  $231 \text{ in}^3$ . What is the volume of a similar bucket that is larger by a scale factor of 3?

$$V = 231 \cdot 3^3 \quad \text{Multiply by the cube of the scale factor.}$$

$$V = 231 \cdot 27 \quad \text{Evaluate the power.}$$

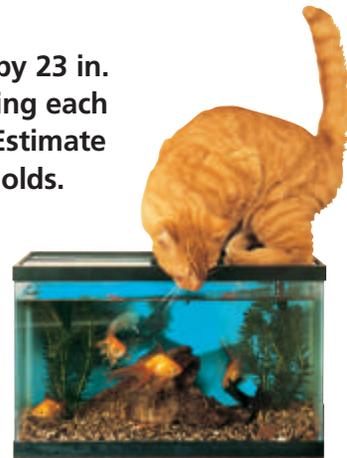
$$V = 6,237 \text{ in}^3 \quad \text{Multiply.}$$

$$\text{Estimate } V \approx 230 \cdot 30 \quad \text{Round the measurements.}$$

$$= 6,900 \text{ in}^3 \quad \text{The answer is reasonable.}$$

**EXAMPLE 3** PROBLEM SOLVING APPLICATION

Elise has a fish tank that measures 10 in. by 23 in. by 5 in. She builds a larger tank by doubling each dimension. There are  $231 \text{ in}^3$  in 1 gallon. Estimate how many more gallons the larger tank holds.

**1 Understand the Problem**

Rewrite the question as a statement.

- Compare the capacities of two similar fish tanks, and estimate how much more water the larger tank holds.

List the **important information**:

- The small tank is 10 in.  $\times$  23 in.  $\times$  5 in.
- The large tank is similar to the small tank by a scale factor of 2.
- $231 \text{ in}^3 = 1 \text{ gal}$

**2 Make a Plan**

You can write an equation that relates the volume of the large tank to the volume of the small tank. Volume of large tank = Volume of small tank  $\cdot$  (scale factor)<sup>3</sup>. Then convert cubic inches to gallons to compare the capacities of the tanks.

**3 Solve**

$$\text{Volume of small tank} = 10 \times 23 \times 5 = 1,150 \text{ in}^3$$

$$\text{Volume of large tank} = 1,150 \cdot 2^3 = 9,200 \text{ in}^3$$

Convert each volume into gallons:

$$1,150 \text{ in}^3 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 5 \text{ gal} \quad 9,200 \text{ in}^3 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 40 \text{ gal}$$

$$\text{Subtract the capacities: } 40 \text{ gal} - 5 \text{ gal} = 35 \text{ gal}$$

The large tank holds about 35 gallons more water than the small tank.

**4 Look Back**

Double the dimensions of the small tank and find the volume:

$$20 \times 46 \times 10 = 9,200 \text{ in}^3. \text{ Subtract the volumes of the two tanks:}$$

$$9,200 - 1,150 = 8,050 \text{ in}^3. \text{ Convert this measurement to gallons:}$$

$$8,050 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 35 \text{ gal}.$$

**Think and Discuss**

- 1. Tell** whether a figure's surface area has increased or decreased if each dimension of the figure is changed by a factor of  $\frac{1}{3}$ .
- 2. Explain** how the surface area of a figure is changed if the dimensions are each multiplied by a factor of 3.
- 3. Explain** how the volume of a figure is changed if the dimensions are each multiplied by a factor of 2.

## GUIDED PRACTICE

- See Example 1
- The surface area of a box is  $10.4 \text{ cm}^2$ . What is the surface area of a similar box that is larger by a scale factor of 3?
  - The surface area of a ship's hull is about  $11,000 \text{ m}^2$ . What is the surface area, to the nearest tenth, of the hull of a model ship that is smaller by a scale factor of  $\frac{1}{150}$ ?
- See Example 2
- The volume of an ice chest is  $2,160 \text{ in}^3$ . What is the volume of a similar ice chest that is larger by a scale factor of 2.5?
- See Example 3
- A fish tank measures 14 in. by 13 in. by 10 in. A similar fish tank is larger by a scale factor of 3. Estimate how many more gallons the larger tank holds.

## INDEPENDENT PRACTICE

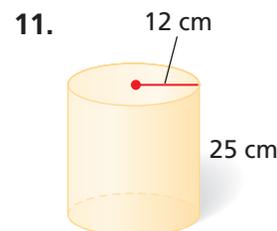
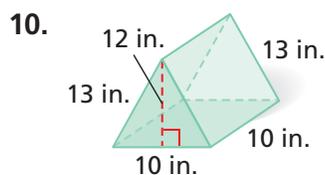
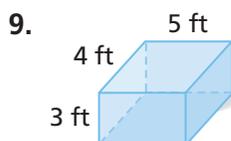
- See Example 1
- The surface area of a triangular prism is  $13.99 \text{ in}^2$ . What is the surface area of a similar prism that is larger by a scale factor of 4?
  - The surface area of a car frame is about  $200 \text{ ft}^2$ . What is the surface area, to the nearest tenth of a square foot, of a model of the car that is smaller by a scale factor of  $\frac{1}{12}$ ?
- See Example 2
- The volume of a cylinder is about  $523 \text{ cm}^3$ . What is the volume, to the nearest tenth, of a similar cylinder that is smaller by a scale factor of  $\frac{1}{4}$ ?
- See Example 3
- A tank measures 27 in. by 9 in. by 12 in. A similar tank is reduced by a scale factor of  $\frac{1}{3}$ . Estimate how many more gallons the larger tank holds.

## PRACTICE AND PROBLEM SOLVING

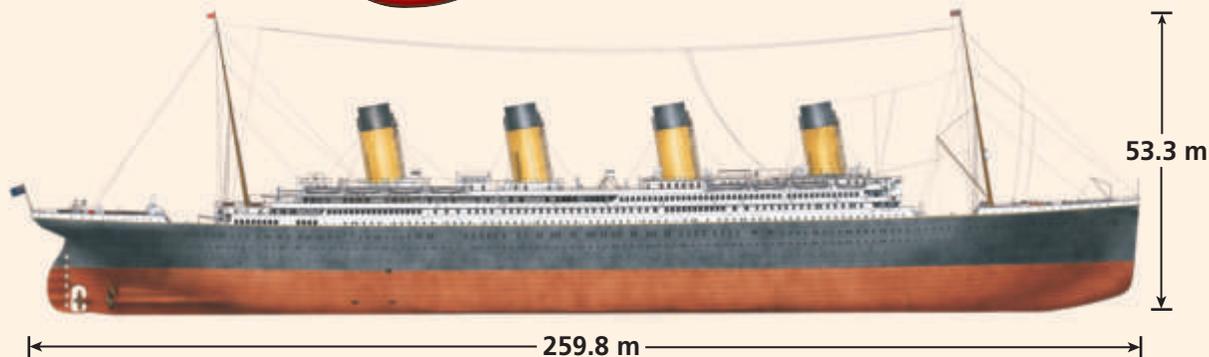
## Extra Practice

See page 747.

For each figure shown, find the surface area and volume of a similar figure that is larger by a scale factor of 25. Use 3.14 for  $\pi$ .



- The surface area of a cylinder is  $1,620 \text{ m}^2$ . Its volume is about  $1,130 \text{ m}^3$ . What are the surface area and volume of a similar cylinder that is smaller by a scale factor of  $\frac{1}{9}$ ? Round to the nearest tenth, if necessary.
- The surface area of a prism is  $142 \text{ in}^2$ . Its volume is about  $105 \text{ in}^3$ . What are the surface area and volume of a similar prism that is larger by a scale factor of 6? Round to the nearest tenth, if necessary.



Natalie and Rebecca are making a scale model of the *Titanic* for a history class project. Their model is smaller by a scale factor of  $\frac{1}{100}$ . For Exercises 14–17, express your answers in both centimeters and meters. Use the conversion chart at right if needed.

**METRIC CONVERSIONS**

$1 \text{ m} = 100 \text{ cm}$	$1 \text{ cm} = 0.01 \text{ m}$
$1 \text{ m}^2 = 10,000 \text{ cm}^2$	$1 \text{ cm}^2 = 0.0001 \text{ m}^2$
$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$	$1 \text{ cm}^3 = 0.000001 \text{ m}^3$

- The length and height of the *Titanic* are shown in the drawing above. What are the length and height of the students' scale model?
- On the students' model, the diameter of the outer propellers is 7.16 cm. What was the diameter of these propellers on the ship?
- The surface area of the deck of the students' model is  $4,156.75 \text{ cm}^2$ . What was the surface area of the deck of the ship?
- The volume of the students' model is about  $127,426 \text{ cm}^3$ . What was the volume of the ship?



These are propellers from the *Olympic*, the *Titanic*'s sister ship. They are identical to those that were on the *Titanic*.


**TEST PREP and Spiral Review**

- Multiple Choice** The surface area of a prism is  $144 \text{ cm}^2$ . A similar prism has a scale factor of  $\frac{1}{4}$ . What is the surface area of the similar prism?  
 (A)  $36 \text{ cm}^2$       (B)  $18 \text{ cm}^2$       (C)  $9 \text{ cm}^2$       (D)  $2.25 \text{ cm}^2$
  - Gridded Response** A cube has a volume of  $64 \text{ in}^3$ . A similar cube has a volume of  $512 \text{ in}^3$ . What is the scale factor of the larger cube?
- Determine whether the ratios are proportional. (Lesson 5-4)
- $\frac{7}{56}, \frac{35}{280}$
  - $\frac{12}{20}, \frac{60}{140}$
  - $\frac{9}{45}, \frac{45}{225}$
  - $\frac{5}{82}, \frac{65}{1,054}$
- Name the polygon that has ten angles and ten sides. (Lesson 8-5)



# Explore Changes in Dimensions



You can use a spreadsheet to explore how changing the dimensions of a rectangular pyramid affects the volume of the pyramid.

## Activity

- On a spreadsheet, enter the following headings:  
*Base Length* in cell A1,  
*Base Width* in cell B1,  
*Height* in cell C1, and  
*Volume* in cell D1.

	A	B	C	D
1	Base Length	Base Width	Height	Volume
2	15	7	22	

In row 2, enter the numbers 15, 7, and 22, as shown.

- Then enter the formula for the volume of a pyramid in cell D2. To do this, enter  $=(1/3)*A2*B2*C2$ . Press **ENTER** and notice that the volume is 770.

	A	B	C	D	E
1	Base Length	Base Width	Height	Volume	
2	15	7	22	$=(1/3)*A2*B2*C2$	

- Enter 30 in cell A2 and 11 in cell C2 to find out what happens to the volume when you double the base length and halve the height.

	A	B	C	D
1	Base Length	Base Width	Height	Volume
2	30	7	11	770

## Think and Discuss

- Explain why doubling the base length and halving the height does not change the volume of the pyramid.
- What other ways could you change the dimensions of the pyramid without changing its volume?

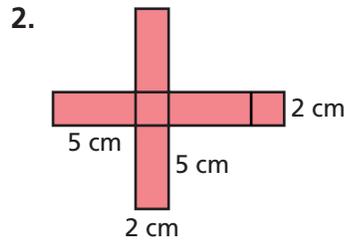
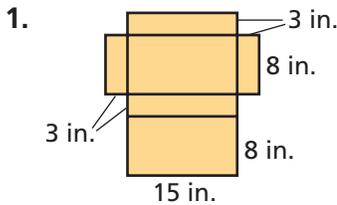
## Try This

- Use a spreadsheet to compute the volume of each cone. Use 3.14 for  $\pi$ .
  - radius = 2.75 inches; height = 8.5 inches
  - radius = 7.5 inches; height = 14.5 inches
- What would the volumes in problem 1 be if the radii were doubled?

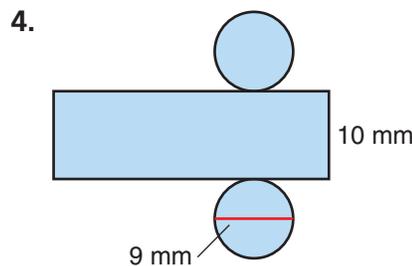
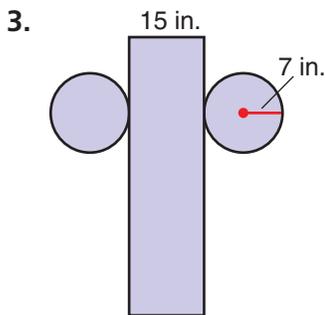
## Quiz for Lessons 10-4 Through 10-5

 **10-4** Surface Area of Prisms and Cylinders

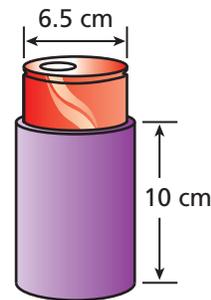
Find the surface area of the prism formed by each net.



Find the surface area of the cylinder formed by each net to the nearest tenth. Use 3.14 for  $\pi$ .



5. The diagram shows a drink can with a drink cooler covering the lower base and part of the curved surface of the can. About how much surface area, in square centimeters, of the drink can is covered by the drink cooler?

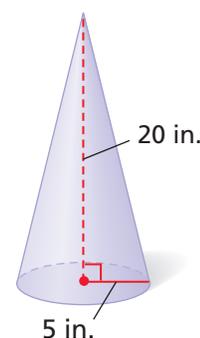

 **10-5** Changing Dimensions

- The surface area of a rectangular prism is  $45 \text{ ft}^2$ . What is the surface area of a similar prism that is larger by a scale factor of 3?
- The surface area of a cylinder is  $109 \text{ cm}^2$ . What is the surface area of a similar cylinder that is smaller by a scale factor of  $\frac{1}{3}$ ?
- The volume of a container is  $3,785 \text{ in}^3$ . A second container is larger by a scale factor of 4. Estimate how many more gallons the larger container holds. (*Hint:* There are  $231 \text{ in}^3$  in 1 gallon.)



**It's a Wrap!** Kim and Miguel are raising money for their school track team by running a gift-wrapping service at the mall. Customers can also have their gifts boxed for shipping. Kim and Miguel have rolls of gift wrap, shipping boxes, cardboard, and packing peanuts.

1. A customer wants to wrap and ship a gift that is in the shape of a rectangular prism. The dimensions of the gift are 10 in. by 15 in. by 4 in. How many square inches of wrapping paper are needed to wrap the gift?
2. Kim chooses a shipping box that is 18 in. by 12 in. by 6 in. After the gift is placed inside the box, she will fill the empty space with packing peanuts. How many cubic inches of packing peanuts will Kim need? Explain.
3. Another customer wants to ship a large cone-shaped art piece made out of recycled glass. The figure shows the dimensions of the conic art. Miguel decides to use poster board to make a cylindrical container that is just large enough to hold the art. How much poster board will he need?
4. Once the conic art is placed in the cylindrical container, how many cubic inches of packing peanuts will be needed to fill the empty space?



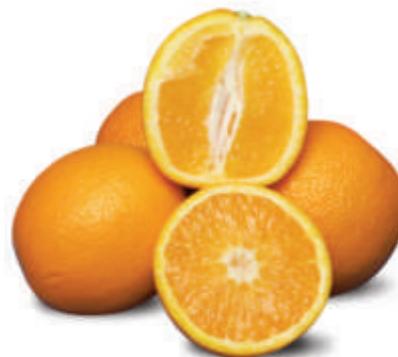
# Cross Sections

**Learn** to sketch and describe cross sections of three-dimensional figures.

## Vocabulary

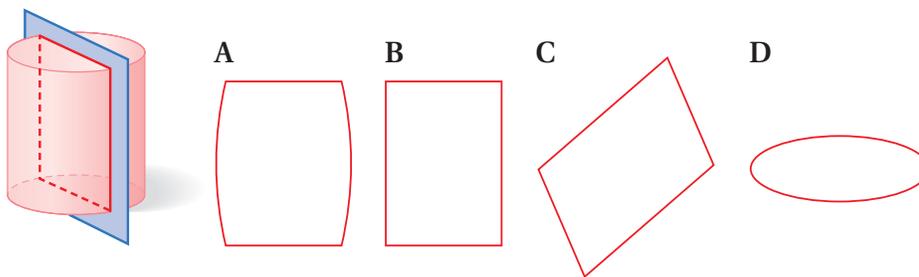
**cross section**

When a three-dimensional figure and a plane intersect, the intersection is called a **cross section**. A three-dimensional figure can have many different cross sections. For example, when you cut an orange in half, the cross section that is exposed depends on the direction of the cut.



### EXAMPLE 1 Identifying Cross Sections

Identify the cross section that best matches the given figure.

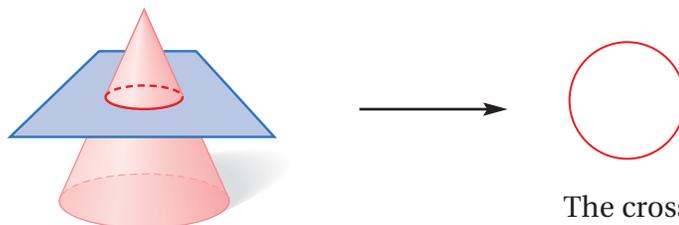


The bases of the cylinder are parallel, so the cross section must contain a pair of parallel lines. The bases of the cylinder meet the lateral surface at right angles, so the cross section must contain right angles. The best choice is B.

### EXAMPLE 2 Sketching and Describing Cross Sections

Sketch and describe the cross section of a cone that is cut parallel to its base.

The base of a cone is a circle. Any cross section made by cutting the cone parallel to the base will also be a circle.



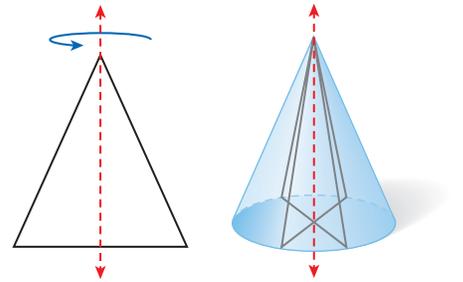
The cross section is a circle.

You can form three-dimensional figures by translating or rotating a cross section through space.

**EXAMPLE 3** Describing Three-Dimensional Figures Formed by Transformations

**Describe the three-dimensional figure formed by rotating an isosceles triangle around its line of symmetry.**

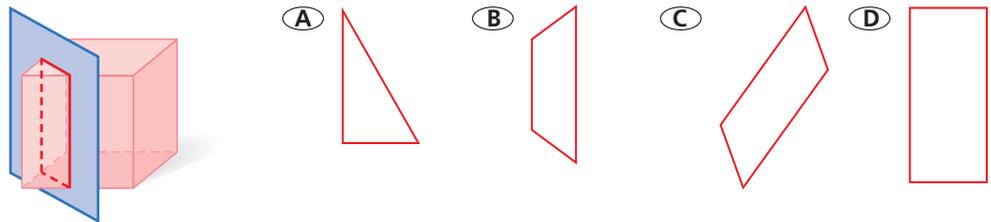
Draw an isosceles triangle and its line of symmetry. Visualize rotating the triangle through space around the line. The resulting three-dimensional figure is a cone.



**EXTENSION**

**Exercises**

1. Identify the cross section that best matches the given figure.



**Sketch and describe each cross section.**

2. a cylinder that is cut parallel to its bases
3. a cube that is cut parallel to one of its faces

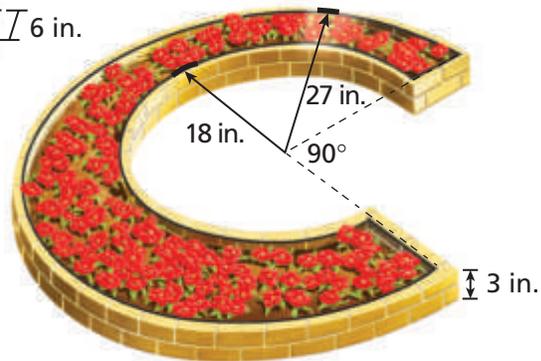
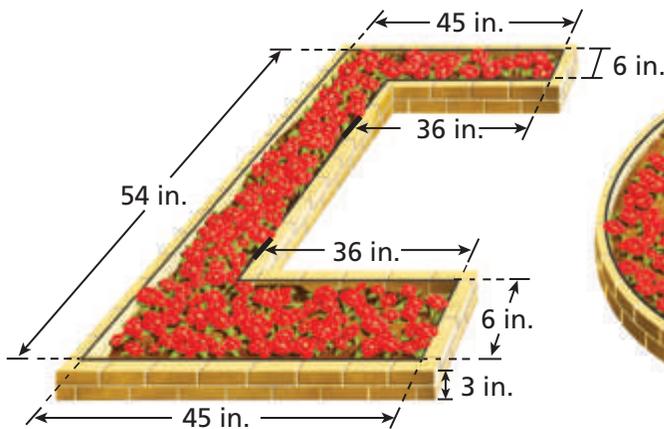
**Describe the three-dimensional figure formed by each transformation.**

4. a rectangle that is rotated around a line of symmetry
5. a circle that is translated perpendicularly to the plane in which it lies  
(*Hint:* Imagine lifting a circle that is lying on a table straight upward.)
6. A sculptor has a block of clay in the shape of a rectangular prism. She uses a piece of wire to cut the clay, and the resulting cross section is a square. Make a sketch showing the prism and how the sculptor may have cut the clay.

# Game Time

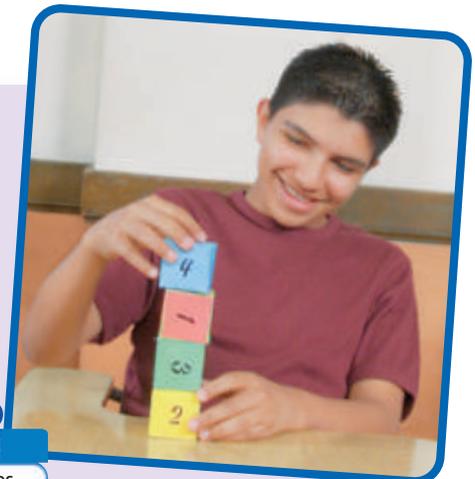
## Blooming Minds

Students in the Agriculture Club at Carter Middle School are designing a flower bed for the front of the school. The flower bed will be in the shape of the letter C. After considering the two designs shown below, the students decided to build the flower bed that required the least amount of peat moss. Which design did the students choose? (*Hint: Find the volume of each flower bed.*)



## Magic Cubes

Four magic cubes are used in this fun puzzle. A complete set of rules and nets for making the cubes can be found online. Each side of the four cubes has the number 1, 2, 3, or 4 written on it. The object of the game is to stack the cubes so that the numbers along each side of the stack add up to 10. No number can be repeated along any side of the stack.



[go.hrw.com](http://go.hrw.com)

Game Time Extra

KEYWORD: MS7 Games



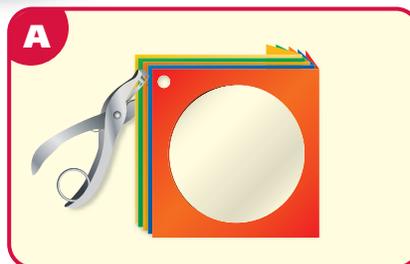
### Materials

- 5 CD envelopes
- hole punch
- chenille stem
- 5 sheets of white paper
- CD
- scissors
- markers

## PROJECT CD 3-D

Make a set of circular booklets that you can store in CD envelopes.

- 1 Stack the CD envelopes so that the flap of each envelope is in the back, along the right-hand edge. Punch a hole through the stack in the upper left-hand corner. **Figure A**
- 2 Insert a chenille stem through the holes, twist to make a loop, and trim the ends. **Figure B**
- 3 Fold a sheet of white  $8\frac{1}{2}$ -by-11-inch paper in half to make a sheet that is  $8\frac{1}{2}$  inches by  $5\frac{1}{2}$  inches. Place the CD on the folded sheet so that it touches the folded edge, and trace around it. **Figure C**
- 4 Cut out the circular shape that you traced, making sure that the two halves remain hinged together. **Figure D**
- 5 Repeat the process with the remaining sheets of paper to make a total of five booklets.



### Taking Note of the Math

Use each booklet to take notes on one lesson of the chapter. Be sure to record essential vocabulary, formulas, and sample problems.



**Vocabulary**

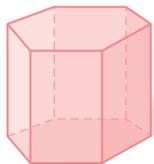
base	580	polyhedron	580
cone	581	prism	580
cylinder	581	pyramid	580
edge	580	surface area	597
face	580	vertex	580
net	597	volume	586

Complete the sentences below with vocabulary words from the list above.

1. A(n) \_\_\_\_?\_\_\_\_ has two parallel, congruent circular bases connected by a curved surface.
2. The sum of the areas of the surfaces of a three-dimensional figure is called the \_\_\_\_?\_\_\_\_.
3. A(n) \_\_\_\_?\_\_\_\_ is a three-dimensional figure whose faces are all polygons.
4. A(n) \_\_\_\_?\_\_\_\_ has one circular base and a curved surface.

**10-1 Introduction to Three-Dimensional Figures** (pp. 580–583)**EXAMPLE**

- Name the figure.

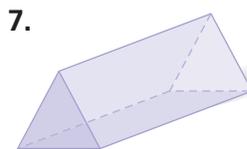
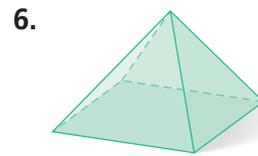
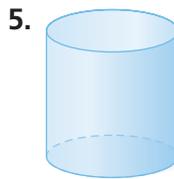


*There are two bases that are hexagons.*

The figure is a hexagonal prism.

**EXERCISES**

Name each figure.



## 10-2 Volume of Prisms and Cylinders (pp. 586–589)

### EXAMPLE

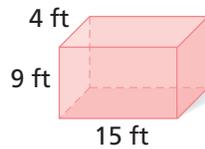
- Find the volume of the prism.

$$V = Bh$$

$$V = (15 \cdot 4) \cdot 9$$

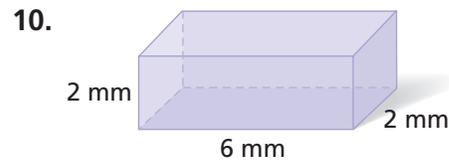
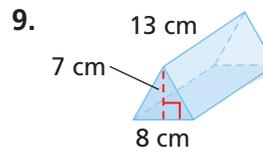
$$V = 540$$

The volume of the prism is  $540 \text{ ft}^3$ .



### EXERCISES

Find the volume of each prism.



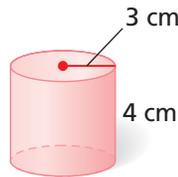
- Find the volume of the cylinder to the nearest tenth. Use 3.14 for  $\pi$ .

$$V = \pi r^2 h$$

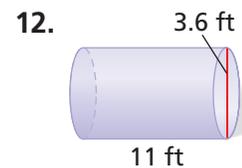
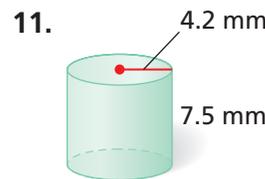
$$V \approx 3.14 \cdot 3^2 \cdot 4$$

$$V \approx 113.04$$

The volume is about  $113.0 \text{ cm}^3$ .



Find the volume of each cylinder to the nearest tenth. Use 3.14 for  $\pi$ .



## 10-3 Volume of Pyramids and Cones (pp. 590–593)

### EXAMPLE

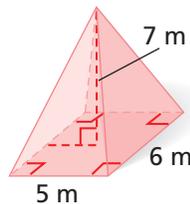
- Find the volume of the pyramid.

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} \cdot (5 \cdot 6) \cdot 7$$

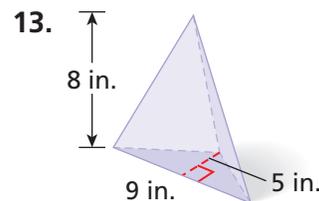
$$V = 70$$

The volume is  $70 \text{ m}^3$ .



### EXERCISES

Find the volume of the pyramid.



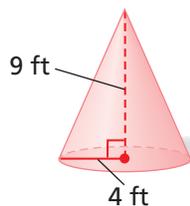
- Find the volume of the cone to the nearest tenth. Use 3.14 for  $\pi$ .

$$V = \frac{1}{3} \pi r^2 h$$

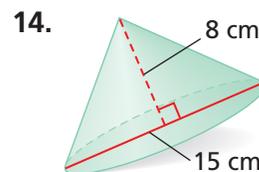
$$V \approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 9$$

$$V \approx 150.72$$

The volume is about  $150.7 \text{ ft}^3$ .



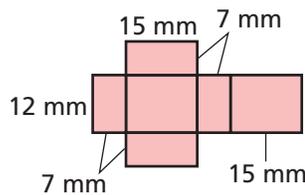
Find the volume of the cone to the nearest tenth. Use 3.14 for  $\pi$ .



## 10-4 Surface Area of Prisms and Cylinders (pp. 597–601)

### EXAMPLE

- Find the surface area of the rectangular prism formed by the net.



$$S = 2\ell w + 2\ell h + 2wh$$

$$S = (2 \cdot 15 \cdot 7) + (2 \cdot 15 \cdot 12) + (2 \cdot 7 \cdot 12)$$

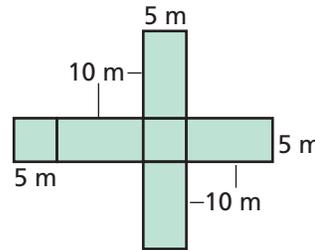
$$S = 738$$

The surface area is  $738 \text{ mm}^2$ .

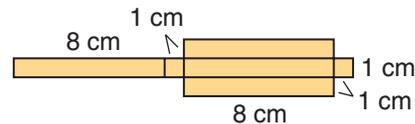
### EXERCISES

Find the surface area of the rectangular prism formed by each net.

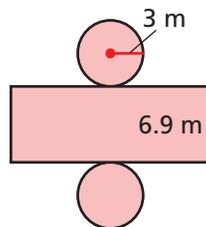
15.



16.



- Find the surface area of the cylinder formed by the net to the nearest tenth.



Use 3.14 for  $\pi$ .

$$S = 2\pi r^2 + 2\pi rh$$

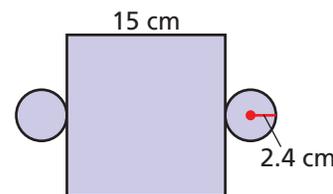
$$S \approx (2 \cdot 3.14 \cdot 3^2) + (2 \cdot 3.14 \cdot 3 \cdot 6.9)$$

$$S \approx 186.516$$

The surface area is about  $186.5 \text{ m}^2$ .

Find the surface area of the cylinder formed by the net to the nearest tenth. Use 3.14 for  $\pi$ .

17.



## 10-5 Changing Dimensions (pp. 604–608)

### EXAMPLE

- The surface area of a rectangular prism is  $32 \text{ m}^2$ , and its volume is  $12 \text{ m}^3$ . What are the surface area and volume of a similar rectangular prism that is larger by a scale factor of 6?

$$S = 32 \cdot 6^2$$

$$= 1,152$$

$$V = 12 \cdot 6^3$$

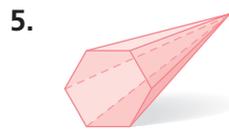
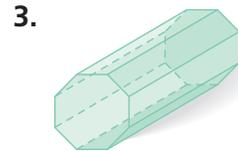
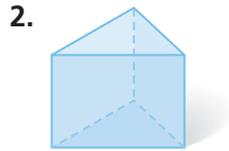
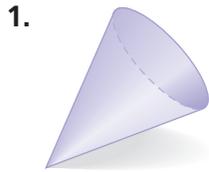
$$= 2,592$$

The surface area of the larger prism is  $1,152 \text{ m}^2$ . Its volume is  $2,592 \text{ m}^3$ .

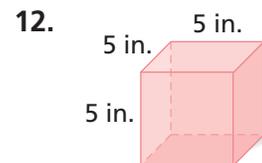
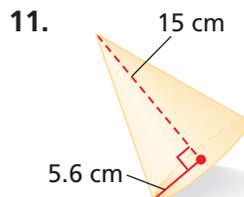
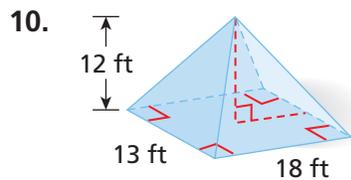
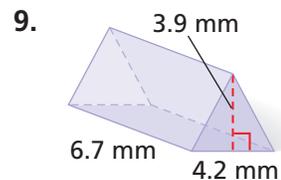
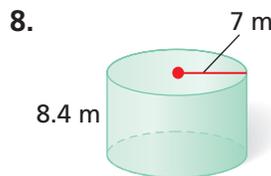
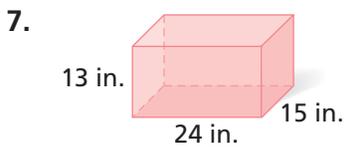
### EXERCISES

- A cylinder has a surface area of  $13.2 \text{ in}^2$ . What is the surface area of a similar cylinder that is larger by a scale factor of 15?
- A refrigerator has a volume of  $14 \text{ ft}^3$ . What is the volume, to the nearest tenth, of a similar refrigerator that is smaller by a scale factor of  $\frac{2}{3}$ ?

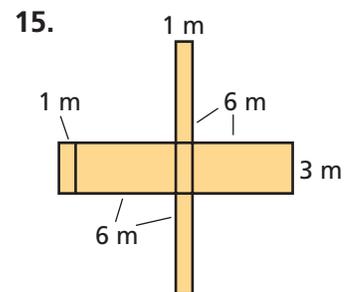
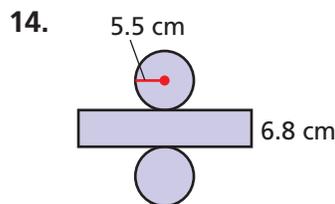
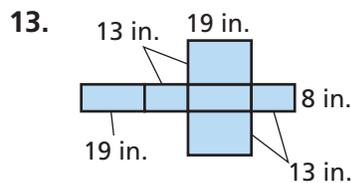
Classify each figure as a polyhedron or not a polyhedron. Then name the figure.



Find the volume of each figure to the nearest tenth. Use 3.14 for  $\pi$ .



Find the surface area of each figure to the nearest tenth. Use 3.14 for  $\pi$ .

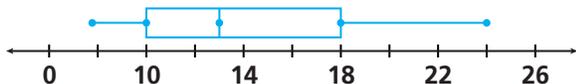


16. The surface area of a rectangular prism is  $52 \text{ ft}^2$ . What is the surface area of a similar prism that is larger by a scale factor of 7?
17. The volume of a cube is  $35 \text{ mm}^3$ . What is the volume of a similar cube that is larger by a scale factor of 9?
18. The volume of a flowerpot is  $7.5 \text{ cm}^3$ . What is the volume, to the nearest hundredth, of a similar flowerpot that is smaller by a scale factor of  $\frac{1}{2}$ ?

**Cumulative Assessment, Chapters 1–10**

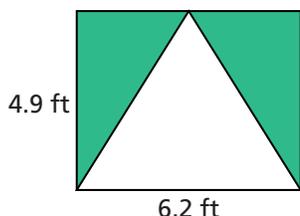
**Multiple Choice**

1. What value represents the median of the data set?



- (A) 7                      (C) 13  
(B) 10                     (D) 18

2. How much less is the area of the triangle than the area of the rectangle?



- (F) 15.19 ft<sup>2</sup>            (H) 9.61 ft<sup>2</sup>  
(G) 30.38 ft<sup>2</sup>            (J) Not here

3. A rectangular tank has a height of 9 meters, a width of 5 meters, and a length of 12 meters. What is the volume of the tank?

- (A) 540 m<sup>3</sup>                (C) 45 m<sup>2</sup>  
(B) 180 m<sup>3</sup>                (D) 26 m<sup>2</sup>

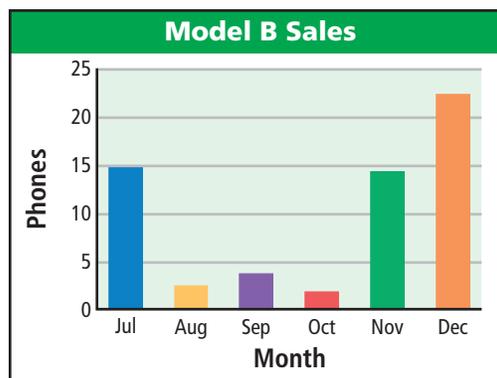
4. What is 140% of 85?

- (F) 11.9                    (H) 1,190  
(G) 119                     (J) 11,900

5. Clay jumps rope at an average rate of 75 jumps per minute. How long does it take him to make 405 jumps if he does not stop?

- (A) 5 min                    (C) 5 $\frac{2}{5}$  min  
(B) 5 $\frac{1}{10}$  min                (D) 5 $\frac{5}{6}$  min

6. A cell-phone company is tracking the sales of a particular model of phone. The sales at one store over six months are shown in the graph. What is the approximate percent increase in sales of Model B phones from October to November?

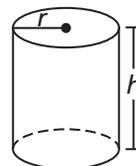


- (F) 40%                     (H) 4,000%  
(G) 400%                   (J) 4%

7. What is the decimal equivalent of  $4\frac{4}{5}$ ?

- (A) 4.45                    (C) 4.8  
(B) 4.54                    (D) 24.5

8. The circumference of the given cylinder is 6 in. What additional information is needed to find the volume of the cylinder?



- (F) diameter                (H) height  
(G) area of base            (J) radius

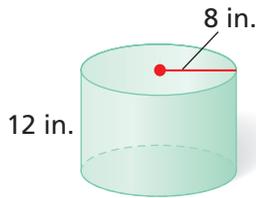
9. The straw has a diameter of 0.6 cm. What is the surface area of the straw?

- (A)  $11.7 \text{ cm}^2$   
 (B)  $5.5 \text{ cm}^2$   
 (C)  $37.3 \text{ cm}^2$   
 (D)  $36.7 \text{ cm}^2$

19.5 cm



10. Find the volume of the cylinder to the nearest tenth. Use 3.14 for  $\pi$ .



- (F)  $602.9 \text{ in}^3$       (H)  $1,205.8 \text{ in}^3$   
 (G)  $3,215.4 \text{ in}^3$       (J)  $2,411.5 \text{ in}^3$



Make sure you use the correct units of measure in your responses. Area has square units, and volume has cubic units.

### Gridded Response

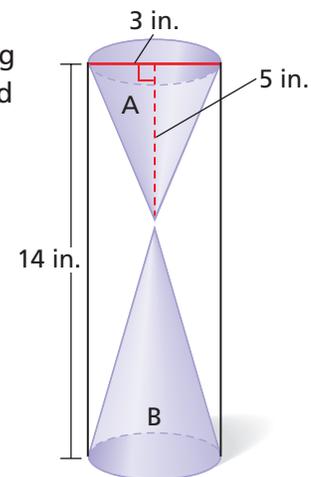
11. A cone-shaped cup has a radius of 4 in. and a volume of  $256 \text{ in}^3$ . What is the height, in inches, of the cup? Round your answer to the nearest tenth.
12. The legs of a right triangle measure 9 units and 12 units. How many units long is the hypotenuse?
13. What is the greatest common factor of 180, 16, and 48?
14. What is the smallest whole-number value of  $x$  that makes the value of the expression  $-15x + 30$  greater than 0?
15. Angle  $A$  and angle  $B$  are vertical angles. If angle  $A$  measures  $62^\circ$ , what is the degree measure of angle  $B$ ?

### Short Response

16. The surface area of a cylinder is  $66 \text{ ft}^2$ .
- Find the surface area of a larger similar cylinder with a scale factor of 4.
  - Explain how the surface area changes when the dimensions are decreased by a factor of  $\frac{1}{4}$ .
17. A polyhedron has two parallel square bases with edges 9 meters long and a height of 9 meters. Identify the figure and find its volume. Show your work.
18. What is the base length of a parallelogram with a height of 8 in. and an area of  $56 \text{ in}^2$ ?

### Extended Response

19. Use the figure for the following problems. Round your answers to the nearest hundredth, if necessary. Use 3.14 for  $\pi$ .

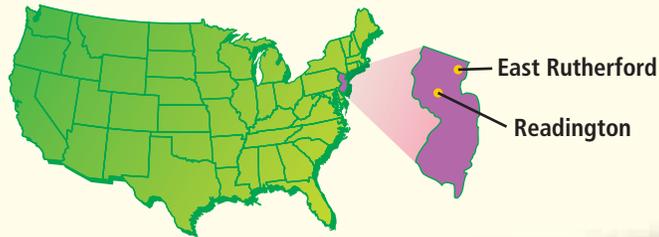


- What three-dimensional shapes make up the sculpture?
- What is the combined volume of figures A and B? Show your work.
- What is the volume of the space surrounding figures A and B? Show your work and explain your answer.



## Problem Solving on Location

### NEW JERSEY



### ★ The Meadowlands Sports Complex

In the early 1970s, construction began on a sports and entertainment facility in East Rutherford, New Jersey. Today, the 700-acre Meadowlands Sports Complex includes a football stadium, an arena, and a racetrack. The complex attracts more than 6 million visitors per year.

Choose one or more strategies to solve each problem.

1. The Meadowlands' stadium and arena both have rectangular video screens. The length of the arena's screen is 36 feet less than the length of the stadium's screen. The stadium's screen has an area of 1,392 square feet and a height of 24 feet. What is the length of the arena's screen?

For 2 and 3, use the table.

2. Rectangular tarps protect the football field during bad weather. Each sheet is 60 feet long and 40 feet wide. How many sheets are needed to cover the field without overlap?
3. For a special presentation, a narrow red carpet is placed around the perimeter of the basketball court. The carpet is also placed diagonally from each corner of the court to the opposite corner. To the nearest foot, how much carpeting is needed?



Court and Field Dimensions		
	Length (ft)	Width (ft)
Football Field	360	160
Basketball Court	94	50



## Problem Solving Strategies

- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Act It Out
- Make an Organized List

# ★ The New Jersey Festival of Ballooning

Every summer, the skies over Readington, New Jersey, are filled with gigantic birthday cakes, sneakers, and polar bears. This unusual sight is part of the New Jersey Festival of Ballooning, a three-day event featuring more than 100 hot air balloons from all over the country.

Choose one or more strategies to solve each problem.

1. The 2005 festival included a balloon shaped like a rectangular prism. The balloon weighed 10 pounds for each foot of its height. The balloon's length was 78 feet, its width was 29 feet, and its volume was 119,886 cubic feet. How much did the balloon weigh?
2. A clown balloon flew at an altitude of 980 feet. A dragon balloon flew 620 feet below the clown balloon. A race car balloon cruised at an altitude of 530 feet. What was the vertical distance between the dragon and the race car? Which balloon was higher?
3. Only one of the balloons shown on the graph came from New Jersey. Use the graph and the following information to determine which balloon came from New Jersey.
  - The height of the balloon from New Jersey was greater than the median height of the balloons.
  - The balloon shaped like an eagle came from Wisconsin.
  - The balloon with a height of 112 feet came from Arizona.

